

Wireless Power Transfer via Strongly Coupled Magnetic Resonances

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Using self-resonant coils in a strongly coupled regime, we experimentally demonstrate efficient non-radiative power transfer over distances of up to eight times the radius of the coils. We demonstrate the ability to transfer 60W with approximately 40% efficiency over distances in excess of two meters. We present a quantitative model describing the power transfer which matches the experimental results to within 5%. We discuss practical applicability and suggest directions for further studies.

In the early 20th century, before the electrical-wire grid, Nikola Tesla (*1*) devoted much effort towards schemes to transport power wirelessly. However, typical embodiments (e.g. Tesla coils) involved undesirably large electric fields. During the past decade, society has witnessed a dramatic surge of use of autonomous electronic devices (laptops, cell-phones, robots, PDAs, etc.) As a consequence, interest in wireless power has re-emerged (*2–4*). Radiative transfer (*5*), while perfectly suitable for transferring information, poses a number of difficulties for power transfer applications: the efficiency of power transfer is very low if the radiation is omnidirectional, and requires an uninterrupted line of sight and sophisticated tracking mechanisms if radiation is unidirectional. A recent theoretical paper (*6*) presented a detailed analysis of the feasibility of using resonant objects coupled through the tails of their non-radiative fields for mid-range energy transfer (*7*). Intuitively, two resonant objects of the same resonant frequency tend to exchange energy efficiently, while interacting weakly with extraneous off-resonant objects. In systems of coupled resonances (e.g. acoustic, electro-magnetic, magnetic, nuclear, etc.), there is often a general “strongly coupled” regime of operation (*8*). If one can operate in that regime in a given system, the energy transfer is expected to be very efficient. Mid-range power transfer implemented this way can be nearly omnidirectional and efficient, irrespective of the geometry of the surrounding space, and with low interference and losses into environmental objects (*6*).

Considerations above apply irrespective of the physical nature of the resonances. In the current work, we focus on one particular physical embodiment: magnetic resonances (*9*). Magnetic resonances are particularly suitable for everyday applications because most of the common materials do not interact with magnetic fields, so interactions with environmental objects are suppressed even further. We were able to identify the strongly coupled regime in the system of two coupled magnetic resonances, by exploring non-radiative (near-field) magnetic resonant induction at MHz frequencies.

At first glance, such power transfer is reminiscent of the usual magnetic induction (*10*); however, note that the usual non-resonant induction is very inefficient for mid-range applications.

Overview of the formalism. Efficient mid-range power transfer occurs in particular regions of the parameter space describing resonant objects strongly coupled to one another. Using coupled-mode theory to describe this physical system (*11*), we obtain the following set of linear equations

$$\dot{a}_m(t) = (i\omega_m - \Gamma_m) a_m(t) + \sum_{n \neq m} i\kappa_{mn} a_n(t) + F_m(t) \quad (1)$$

where the indices denote the different resonant objects. The variables $a_m(t)$ are defined so that the energy contained in object m is $|a_m(t)|^2$, ω_m is the resonant frequency of that isolated object, and Γ_m is its intrinsic decay rate (e.g. due to absorption and radiated losses), so that in this framework an uncoupled and undriven oscillator with parameters ω_0 and Γ_0 would evolve in time as $\exp(i\omega_0 t - \Gamma_0 t)$. The $\kappa_{mn} = \kappa_{nm}$ are coupling coefficients between the resonant objects indicated by the subscripts, and $F_m(t)$ are driving terms.

We limit the treatment to the case of two objects, denoted by source and device, such that the source (identified by the subscript S) is driven externally at a constant frequency, and the two objects have a coupling coefficient κ . Work is extracted from the device (subscript D) by means of a load (subscript W) which acts as a circuit resistance connected to the device, and has the effect of contributing an additional term Γ_W to the unloaded device object's decay rate Γ_D . The overall decay rate at the device is therefore $\Gamma'_D = \Gamma_D + \Gamma_W$. The work extracted is determined by the power dissipated in the load, i.e. $2\Gamma_W |a_D(t)|^2$. Maximizing the efficiency η of the transfer with respect to the loading Γ_W , given Eq. 1, is equivalent to solving an impedance matching problem. One finds that the scheme works best when the source and the device are resonant, in which case the efficiency is

$$\begin{aligned} \eta &= \frac{\Gamma_W |a_D|^2}{\Gamma_S |a_S|^2 + (\Gamma_D + \Gamma_W) |a_D|^2} \\ &= \frac{(\Gamma_W/\Gamma_D)\kappa^2/(\Gamma_S\Gamma_D)}{(1 + \Gamma_W/\Gamma_D)\kappa^2/(\Gamma_S\Gamma_D) + (1 + \Gamma_W/\Gamma_D)^2} \quad (2) \end{aligned}$$

The efficiency is maximized when $\Gamma_W/\Gamma_D = (1 + \kappa^2/\Gamma_S\Gamma_D)^{1/2}$. It is easy to show that the key to efficient energy transfer is to have $\kappa^2/\Gamma_S\Gamma_D > 1$. This is commonly referred to as the strong coupling regime. Resonance plays an essential role in this

power transfer mechanism, as the efficiency is improved by approximately ω^2/Γ_D^2 ($\sim 10^6$ for typical parameters) compared to the case of inductively coupled non-resonant objects.

Theoretical model for self-resonant coils. Our experimental realization of the scheme consists of two self-resonant coils, one of which (the source coil) is coupled inductively to an oscillating circuit, while the other (the device coil) is coupled inductively to a resistive load (*I2*) (Fig. 1). Self-resonant coils rely on the interplay between distributed inductance and distributed capacitance to achieve resonance. The coils are made of an electrically conducting wire of total length l and cross-sectional radius a wound into a helix of n turns, radius r , and height h . To the best of our knowledge, there is no exact solution for a finite helix in the literature, and even in the case of infinitely long coils, the solutions rely on assumptions that are inadequate for our system (*I3*). We have found, however, that the simple quasi-static model described below is in good agreement (approximately 5%) with experiment.

We start by observing that the current has to be zero at the ends of the coil, and make the educated guess that the resonant modes of the coil are well approximated by sinusoidal current profiles along the length of the conducting wire. We are interested in the lowest mode, so if we denote by s the parameterization coordinate along the length of the conductor, such that it runs from $-l/2$ to $+l/2$, then the time-dependent current profile has the form $I_0 \cos(\pi s/l) \exp(i\omega t)$. It follows from the continuity equation for charge that the linear charge density profile is of the form $\lambda_0 \sin(\pi s/l) \exp(i\omega t)$, so the two halves of the coil (when sliced perpendicularly to its axis) contain charges equal in magnitude $q_0 = \lambda_0 l/\pi$ but opposite in sign.

As the coil is resonant, the current and charge density profiles are $\pi/2$ out of phase from each other, meaning that the real part of one is maximum when the real part of the other is zero. Equivalently, the energy contained in the coil is at certain points in time completely due to the current, and at other points, completely due to the charge. Using electromagnetic theory, we can define an effective inductance L and an effective capacitance C for each coil as follows:

$$L = \frac{\mu_0}{4\pi |I_0|^2} \iint d\mathbf{r} d\mathbf{r}' \frac{\mathbf{J}(\mathbf{r}) \cdot \mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \quad (3)$$

$$\frac{1}{C} = \frac{1}{4\pi \epsilon_0 |q_0|^2} \iint d\mathbf{r} d\mathbf{r}' \frac{\rho(\mathbf{r})\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \quad (4)$$

where the spatial current $\mathbf{J}(\mathbf{r})$ and charge density $\rho(\mathbf{r})$ are obtained respectively from the current and charge densities along the isolated coil, in conjunction with the geometry of the object. As defined, L and C have the property that the energy U contained in the coil is given by

$$\begin{aligned} U &= \frac{1}{2} L |I_0|^2 \\ &= \frac{1}{2C} |q_0|^2 \end{aligned} \quad (5)$$

Given this relation and the equation of continuity, one finds that the resonant frequency is $f_0 = 1/2\pi[(LC)^{1/2}]$. We can now treat this coil as a standard oscillator in coupled-mode theory by defining $a(t) = [(L/2)^{1/2}]I_0(t)$.

We can estimate the power dissipated by noting that the sinusoidal profile of the current distribution implies that the spatial average of the peak current-squared is $|I_0|^2/2$. For a coil with n turns and made of a material with conductivity σ , we modify the standard formulas for ohmic (R_o) and radiation (R_r) resistance accordingly:

$$R_o = \sqrt{\frac{\mu_0 \omega}{2\sigma}} \frac{l}{4\pi a} \quad (6)$$

$$R_r = \sqrt{\frac{\mu_0}{\epsilon_0}} \left(\frac{\pi}{12} n^2 \left(\frac{\omega r}{c} \right)^4 + \frac{2}{3\pi^3} \left(\frac{\omega h}{c} \right)^2 \right) \quad (7)$$

The first term in Eq. 7 is a magnetic dipole radiation term (assuming $r \ll 2\pi c/\omega$); the second term is due to the electric dipole of the coil, and is smaller than the first term for our experimental parameters. The coupled-mode theory decay constant for the coil is therefore $\Gamma = (R_o + R_r)/2L$, and its quality factor is $Q = \omega/2\Gamma$.

We find the coupling coefficient κ_{DS} by looking at the power transferred from the source to the device coil, assuming a steady-state solution in which currents and charge densities vary in time as $\exp(i\omega t)$.

$$\begin{aligned} P_{DS} &= \int d\mathbf{r} \mathbf{E}_S(\mathbf{r}) \cdot \mathbf{J}_D(\mathbf{r}) \\ &= - \int d\mathbf{r} (\dot{\mathbf{A}}_S(\mathbf{r}) + \nabla \phi_S(\mathbf{r})) \cdot \mathbf{J}_D(\mathbf{r}) \\ &= - \frac{1}{4\pi} \iint d\mathbf{r} d\mathbf{r}' \left(\mu_0 \frac{\mathbf{J}_S(\mathbf{r}')}{|\mathbf{r}' - \mathbf{r}|} + \frac{\rho_S(\mathbf{r}')}{\epsilon_0} \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^3} \right) \cdot \mathbf{J}_D(\mathbf{r}') \\ &\equiv -i\omega M I_S I_D \end{aligned} \quad (8)$$

where the subscript S indicates that the electric field is due to the source. We then conclude from standard coupled-mode theory arguments that $\kappa_{DS} = \kappa_{SD} = \kappa = \omega M/2[(L_S L_D)^{1/2}]$. When the distance D between the centers of the coils is much larger than their characteristic size, κ scales with the D^{-3} dependence characteristic of dipole-dipole coupling. Both κ and Γ are functions of the frequency, and κ/Γ and the efficiency are maximized for a particular value of f , which is in the range 1–50MHz for typical parameters of interest. Thus, picking an appropriate frequency for a given coil size, as we do in this experimental demonstration, plays a major role in optimizing the power transfer.

Comparison with experimentally determined parameters. The parameters for the two identical helical coils built for the experimental validation of the power transfer scheme are $h = 20\text{cm}$, $a = 3\text{mm}$, $r = 30\text{cm}$, and $n = 5.25$. Both coils are made of copper. The spacing between loops of the helix is not uniform, and we encapsulate the uncertainty about their uniformity by attributing a 10% (2cm) uncertainty to h . The expected resonant frequency given these

dimensions is $f_0 = 10.56 \pm 0.3\text{MHz}$, which is about 5% off from the measured resonance at 9.90MHz.

The theoretical Q for the loops is estimated to be approximately 2500 (assuming $\sigma = 5.9 \times 10^7 \text{ m}/\Omega$) but the measured value is $Q = 950 \pm 50$. We believe the discrepancy is mostly due to the effect of the layer of poorly conducting copper oxide on the surface of the copper wire, to which the current is confined by the short skin depth ($\sim 20\mu\text{m}$) at this frequency. We therefore use the experimentally observed Q and $\Gamma_S = \Gamma_D = \Gamma = \omega/2Q$ derived from it in all subsequent computations.

We find the coupling coefficient κ experimentally by placing the two self-resonant coils (fine-tuned, by slightly adjusting h , to the same resonant frequency when isolated) a distance D apart and measuring the splitting in the frequencies of the two resonant modes. According to coupled-mode theory, this splitting should be $\Delta\omega = 2[(\kappa^2 - \Gamma^2)^{1/2}]$. In the present work, we focus on the case where the two coils are aligned coaxially (Fig. 2), although similar results are obtained for other orientations (figs. S1 and S2).

Measurement of the efficiency. The maximum theoretical efficiency depends only on the parameter $\kappa/[(L_S L_D)^{1/2}] = \kappa/\Gamma$, which is greater than 1 even for $D = 2.4\text{m}$ (eight times the radius of the coils) (Fig. 3), thus we operate in the strongly-coupled regime throughout the entire range of distances probed.

As our driving circuit, we use a standard Colpitts oscillator whose inductive element consists of a single loop of copper wire 25cm in radius (Fig. 1); this loop of wire couples inductively to the source coil and drives the entire wireless power transfer apparatus. The load consists of a calibrated light-bulb (14), and is attached to its own loop of insulated wire, which is placed in proximity of the device coil and inductively coupled to it. By varying the distance between the light-bulb and the device coil, we are able to adjust the parameter Γ_w/Γ so that it matches its optimal value, given theoretically by $(1 + \kappa^2/\Gamma^2)^{1/2}$. (The loop connected to the light-bulb adds a small reactive component to Γ_w which is compensated for by slightly retuning the coil.) We measure the work extracted by adjusting the power going into the Colpitts oscillator until the light-bulb at the load glows at its full nominal brightness.

We determine the efficiency of the transfer taking place between the source coil and the load by measuring the current at the mid-point of each of the self-resonant coils with a current-probe (which does not lower the Q of the coils noticeably.) This gives a measurement of the current parameters I_S and I_D used in our theoretical model. We then compute the power dissipated in each coil from $P_{S,D} = \Gamma L |I_{S,D}|^2$, and obtain the efficiency from $\eta = P_w/(P_S + P_D + P_w)$. To ensure that the experimental setup is well described by a two-object coupled-mode theory model, we position the device coil such that its direct coupling to the copper loop attached to the Colpitts oscillator is zero. The experimental results are shown in Fig. 4, along with the theoretical prediction for maximum efficiency, given by Eq. 2. We are able to transfer significant amounts of power using this setup, fully lighting up a 60W light-bulb from distances more than 2m away (figs. S3 and S4).

As a cross-check, we also measure the total power going from the wall power outlet into the driving circuit. The efficiency of the wireless transfer itself is hard to estimate in

this way, however, as the efficiency of the Colpitts oscillator itself is not precisely known, although it is expected to be far from 100% (15). Still, the ratio of power extracted to power entering the driving circuit gives a lower bound on the efficiency. When transferring 60W to the load over a distance of 2m, for example, the power flowing into the driving circuit is 400W. This yields an overall wall-to-load efficiency of 15%, which is reasonable given the expected efficiency of roughly 40% for the wireless power transfer at that distance and the low efficiency of the Colpitts oscillator.

Concluding remarks. It is essential that the coils be on resonance for the power transfer to be practical (6). We find experimentally that the power transmitted to the load drops sharply as either one of the coils is detuned from resonance. For a fractional detuning $\Delta f/f_0$ of a few times the inverse loaded Q , the induced current in the device coil is indistinguishable from noise.

A detailed and quantitative analysis of the effect of external objects on our scheme is beyond the scope of the current work, but we would like to note here that the power transfer is not visibly affected as humans and various everyday objects, such as metals, wood, and electronic devices large and small, are placed between the two coils, even in cases where they completely obstruct the line of sight between source and device (figs. S3 to S5). External objects have a noticeable effect only when they are within a few centimeters from either one of the coils. While some materials (such as aluminum foil, styrofoam and humans) mostly just shift the resonant frequency, which can in principle be easily corrected with a feedback circuit, others (cardboard, wood, and PVC) lower Q when placed closer than a few centimeters from the coil, thereby lowering the efficiency of the transfer.

When transferring 60W across 2m, we calculate that at the point halfway between the coils the RMS magnitude of the electric field is $E_{rms} = 210\text{V/m}$, that of the magnetic field is $H_{rms} = 1\text{A/m}$, and that of the Poynting vector is $S_{rms} = 3.2\text{mW/cm}^2$ (16). These values increase closer to the coils, where the fields at source and device are comparable. For example, at distances 20cm away from the surface of the device coil, we calculate the maximum values for the fields to be $E_{rms} = 1.4\text{kV/m}$, $H_{rms} = 8\text{A/m}$, and $S_{rms} = 0.2\text{W/cm}^2$. The power radiated for these parameters is approximately 5W, which is roughly an order of magnitude higher than cell phones. In the particular geometry studied in this article, the overwhelming contribution (by one to two orders of magnitude) to the electric near-field, and hence to the near-field Poynting vector, comes from the electric dipole moment of the coils. If instead one uses capacitively-loaded single-turn loop design (6) - which has the advantage of confining nearly all of the electric field inside the capacitor - and tailors the system to operate at lower frequencies, our calculations show (17) that it should be possible to reduce the values cited above for the electric field, the Poynting vector, and the power radiated to below general safety regulations (e.g. the IEEE safety standards for general public exposure (18).)

Although the two coils are currently of identical dimensions, it is possible to make the device coil small enough to fit into portable devices without decreasing the efficiency. One could, for instance, maintain the product of the characteristic sizes of the source and device coils constant, as argued in (6).

We believe that the efficiency of the scheme and the power transfer distances could be appreciably improved by silver-plating the coils, which should increase their Q , or by working with more elaborate geometries for the resonant objects (19). Nevertheless, the performance characteristics of the system presented here are already at levels where they could be useful in practical applications.

References and Notes

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20. The authors would like to thank John Pendry for suggesting the use of magnetic resonances, and Michael Grossman and Ivan Čelanović for technical assistance. This work was supported in part by the Materials Research Science and Engineering Center program of the National Science Foundation under Grant No. DMR 02-13282, by the U.S. Department of Energy under Grant No. DE-FG02-99ER45778, and by the Army Research Office through the Institute for Soldier Nanotechnologies under Contract No. DAAD-19-02-D0002.

Supporting Online Material

www.sciencemag.org/cgi/content/full/1143254/DC1
SOM Text
Figs. S1 to S5

30 March 2007; accepted 21 May 2007

Published online 7 June 2007; 10.1126/science.1143254

Include this information when citing this paper.

Fig. 1. Schematic of the experimental setup. A is a single copper loop of radius 25cm that is part of the driving circuit, which outputs a sine wave with frequency 9.9MHz. S and D are respectively the source and device coils referred to in the text. B is a loop of wire attached to the load (“light-bulb”). The various κ 's represent direct couplings between the objects indicated by the arrows. The angle between coil D and the loop A is adjusted to ensure that their direct coupling is zero, while coils S and D are aligned coaxially. The direct couplings between B and A and between B and S are negligible.

Fig. 2. Comparison of experimental and theoretical values for κ as a function of the separation between coaxially aligned source and device coils (the wireless power transfer distance.)

Fig. 3. Comparison of experimental and theoretical values for the parameter κ/Γ as a function of the wireless power transfer distance. The theory values are obtained by using the theoretical κ and the experimentally measured Γ . The shaded area represents the spread in the theoretical κ/Γ due to the 5% uncertainty in Q .

Fig. 4. Comparison of experimental and theoretical efficiencies as functions of the wireless power transfer distance. The shaded area represents the theoretical prediction for maximum efficiency, and is obtained by inserting the theoretical values from Fig. 3 into Eq. 2 [with $\Gamma_w/\Gamma_D = (1 + \kappa^2/\Gamma^2)^{1/2}$]. The black dots are the maximum efficiency obtained from Eq. 2 and the experimental values of κ/Γ from Fig. 3. The red dots present the directly measured efficiency, as described in the text.







