

How far are we from the quantum theory of gravity?

Lee Smolin

Perimeter Institute for Theoretical Physics

Waterloo, Canada

and

Department of Physics, University of Waterloo

lsmolin@perimeterinstitute.ca

March 19, 2003

ABSTRACT

An assessment is offered of the progress that the major approaches to quantum gravity have made towards the goal of constructing a complete and satisfactory theory. The emphasis is on loop quantum gravity and string theory, although other approaches are discussed, including dynamical triangulation models (euclidean and lorentzian) regge calculus models, causal sets, twistor theory, non-commutative geometry and models based on analogies to condensed matter systems. We proceed by listing the questions the theories are expected to be able to answer. We then compile two lists: the first details the actual results so far achieved in each theory, while the second lists conjectures which remain open. By comparing them we can evaluate how far each theory has progressed, and what must still be done before each theory can be considered a satisfactory quantum theory of gravity. We find there has been impressive recent progress on several fronts. At the same time, important issues about loop quantum gravity are so far unresolved, as are key conjectures of string theory. However, there is a reasonable expectation that experimental tests of lorentz invariance at Planck scales may in the near future make it possible to rule out one or more candidate quantum theories of gravity.

Contents

1	Introduction	4
1.1	Methodology	6
2	Physical questions the theories should answer	10
2.1	Questions concerning quantum gravity	10
2.2	Questions concerning cosmology	12
2.3	Questions concerning unification of the forces	13
2.4	Foundational questions	13
3	A tale of two theories	14
3.1	Common postulates	14
3.2	Why string theory and loop quantum gravity differ	15
3.3	Characteristic predictions of string theory	16
3.4	Characteristic predictions of loop quantum gravity.	16
4	The near term experimental situation	17
5	Postulates and main results of loop quantum gravity	19
5.1	Postulates of loop quantum gravity I	20
5.2	The main results of loop quantum gravity I	20
5.3	Loop quantum gravity II	26
5.4	Open questions in loop quantum gravity	27
6	Definition and main results of string theory	30
6.1	The definition of a string theory	30
6.2	Basic results of perturbative string theory	32
6.3	Results and conjectures concerning black holes	36
6.4	Results and conjectures concerning dualities	38
6.4.1	T duality	38
6.4.2	S duality	38
6.4.3	String/gauge theory dualities	39
6.5	Open issues of string theory	45
6.6	Open conjectures of string theory	48
7	Other approaches	51
7.1	Dynamical triangulation models	51
7.2	Regge calculus models	52
7.3	Causal set models	52
7.4	Twistor theory [200]	54
7.5	Non-commutative geometry	54
7.6	Condensed matter physics inspired models	55

8	How well do the theories answer the questions?	56
8.1	Quantum gravity questions	56
8.2	Cosmological questions	58
8.3	Questions concerning unification	59
8.4	Foundational questions	60
9	Conclusions	61
9.1	What remains to be done in loop quantum gravity?	61
9.2	What remains to be done in string theory?	62

1 Introduction

This paper is dedicated to Stanley Deser, Bryce DeWitt, Cecille Morette-DeWitt, David Finkelstein, Chris Isham, Karel Kuchar, Roger Penrose and John Archibald Wheeler, each pioneers who have been and who remain continuing sources of inspiration and encouragement for all of us working in quantum gravity.

For most of the twentieth century physics proceeded with two fundamental physical theories, quantum theory and general relativity. The latter is Einstein's theory of space, time and gravitation, while the former describes essentially everything else in nature. This situation was possible, because there were no experiments that probed regimes in which both quantum and gravitational effects were present. At the same time, the fact that nature is one entity meant that there must eventually be discovered a unification of quantum theory and general relativity, which could stand as a single theory of nature. Such a theory is called a quantum theory of gravity.

Not so many years ago, it was common to hear the statement that there is no quantum theory of gravity and that the invention of such a theory is far off. Although a few people have worked on the problem of quantum gravity since the 1950's, no great progress was made until the early seventies, apart from technical developments which ruled out various approaches. Then, around 1971 several striking results were found, concerning the behavior of quantum fields on a few spacetime backgrounds besides Minkowski spacetime. These included Bekenstein's discovery of the entropy of black holes[1], Hawking's discovery that black holes are hot[2], and radiate, and Unruh's discovery that even the vacuum of flat spacetime behaves as a thermal state when viewed by an accelerating observer[3]. These effects all point to the possibility of a deep connection between spacetime, quantum theory and thermodynamics, which has fascinated people ever since.

Still, this was not quantum gravity, as the geometry of spacetime, and the gravitational field, were still treated as in Einstein's classical theory. Real, undeniable, progress on quantum gravity began only in the mid 1980's. The reason was the almost simultaneous invention of two approaches to quantum gravity which each quickly achieved impressive advances towards the solution of some aspect of quantum gravity. These two developments were string theory[4, 5] and loop quantum gravity[7]-[18].

Since then both string theory and loop quantum gravity have been the subject of a large and intense effort by many people¹. After 18 years, a large number of results have accumulated about each theory. In addition, in recent years several new approaches have been invented, the include causal dynamical triangulations, non-commutative geometry, causal sets and approaches based on analogies to condensed matter physics. The main purpose of this essay is to make an evaluation of where each theory stands in relation to the main questions that a quantum theory of gravity is expected to answer.

¹For popular accounts of string theory and loop quantum gravity, see [6] and [19]. Further pedagogical material is available on several websites[20].

One reason to carry out such an evaluation is that, while the undeniably impressive progress on several sides has generated a lot of excitement among both experts and the wider community, there appears at the same time to be a great deal of confusion about exactly what each of the theories has so far achieved. This is perhaps surprising, as it does not appear to have been the case with earlier theoretical triumphs such as quantum theory or relativity. Still, one only has to talk to a wide enough selection of experts to get the impression that there is quite a lot of disagreement about the significance of the results so far achieved on each side². In some cases there is even confusion about what the actual results are.

This confusion has several sources. The first is the gap that necessarily exists between the highly technical and qualified language that must be used to describe the actual results and the more general language that is used to convey their significance to a wider audience, not only to non-scientists, but to physicists and mathematicians who are not experts in the theory in question. It is also unfortunately true that some, although of course not all, proponents of each theory have sometimes simplified the statements of results in presentations for non-experts in such a way as to appear to claim results which have in fact not yet been shown. There is also a lot of confusion caused by the fact that in a few crucial cases, there are conjectures which are widely believed by experts, in spite of the fact that they remain unproven. Additional confusion comes from the fact that some of these conjectures come in different inequivalent forms.

Another source of confusion is the very unfortunate isolation in which each community works. It is striking that there has never been a talk on loop quantum gravity at one of the annual string theory conferences. And, while there are ongoing conversations between some people in the two communities, there are very few people who have done technical work in both theories. As a result, many experts in one approach have only a very superficial understanding of the other.

The sad result is that many members of each community sincerely believe that the approach they work on is the only viable approach to quantum gravity. This of course causes still more alienation which further separates the two communities. It is very offensive for someone working on loop quantum gravity to listen to a talk or read a book or paper that begins, as they unfortunately often do, with the assertion that “string theory is the only quantum theory of gravity.” At the same time, string theorists listening to talks on loop quantum gravity are often puzzled by the lack of interest in supersymmetry and higher dimensions, which string theory has shown seem to be required to satisfy certain criteria for a good theory³.

For all of these reasons, it seems important to attempt to carry out an objective evaluation of the status of these two research programs. The present paper is an attempt to do just this. It began, indeed, as a personal project, for as someone who has worked on both theories

²See, for example, [21]. For a critical view of string theory by one of its pioneers, see, [22].

³In fact there are results that show that loop quantum gravity extends easily to supergravity, at least through $N = 2$ and there are even partial results on 11 dimensional supergravity[23, 25, 24, 27]. Moreover some results on spin foams extend to $d > 4$ [26].

I found myself in a situation in which I was quite confused and puzzled about the status of each of the two theories. In order to decide which to continue working in, and on what problems, I decided to undertake an analysis of the actual results in each case. By doing so one can see more clearly what would have to be done in each case to move the theory from its present situation to the status of a true physical theory.

1.1 Methodology

The method I decided to pursue is mirrored in the structure of the paper that follows. I began by writing down a list of the questions that the theories are expected to be able to answer. Then I wrote down as complete a list as I could of the major results of the two theories. I also made a list of the conjectures that have been made in the course of the development of the two theories, and for each considered whether it had been demonstrated, or disproven, by the results. Or, if neither was the case, I tried to establish to what extent the actual results could be taken as providing evidence for the conjectures.

There are different standards by which physicists and mathematicians judge the reliability of results. I took as appropriate those of mainstream theoretical physics. I do not require the rigor of mathematical physics, although, as will be pointed out, there are results for each theory that are at this level. When a result is claimed regarding a quantum field theory, it should have been obtained in a context in which all expressions have been regulated, all divergences or ambiguities are resolved, and careful attention has been given to technical issues such as how the gauge invariances of the classical theory are maintained in the quantum theory. When a path integral is involved it should be fully defined in terms of a well defined measure, or else expressed as a discrete summation.

This led in each case to two lists, the first of results, the second of conjectures and open issues. These are summarized in Table 1, which indicates the extent to which each of the theories answers each of the questions posed. After this I asked what steps remain before each theory might be considered complete. By this I meant that it is precisely formulated and well understood mathematically and conceptually, that there are methods to carry out calculations leading to predictions for real experiments, and that at least a few experiments have been done which either support or falsify the predictions of the theory.

Table 1 and the lists of things still to do comprise the main conclusions of this essay. What to do about the present situation, whether to continue to investigate one or the other or both theories is a matter of opinion and an individual's research strategy. I will indicate my own personal conclusions at the end, however I want to stress that I do not believe that all experts will, or even should agree on these questions. Science works best when there is a variety of viewpoints investigated, and when there is room in the community for people who investigate a range of viable approaches to any unsolved problem. But I do think that it is a useful exercise to try at least to come to a consensus about what the evidence is, what has been done and what remains to be done. I hope that this essay will contribute to that goal.

Let me describe some issues which arose in carrying out this program. First, it is important to distinguish two kinds of results. The first are results which further our understanding

of physical questions the theory was originally invented to answer. These are to be distinguished from results which resolve issues and puzzles raised by the theory itself, whose solution will help us understand the theory better, but which will not lead to the answer to a question about nature. Results of the first kind we may call *substantial*, whereas results of the second kind can be called *internal*.

While this is not an ironclad distinction it is a useful one. A lot of progress can be (and often must be) made on understanding the mathematical structure of a theory, without any actual progress being made on any question about the natural world. In evaluating the status of a theory we may be impressed by progress on the second kind of question, but the main focus must be the first kind.

This is especially the case with a complicated theory like string theory, which has many, perhaps an infinite number of, versions of which do not describe the universe we live in. In a case like this we must distinguish between measures of activity, which may result in various aspects of the theory being explored which do not relate, even indirectly, to nature, and measures of progress leading to understanding some feature of the natural world or to new predictions for real, doable experiments.

To distinguish between these two kinds of questions it is important to keep in mind what aspects of nature are known from experiment and which are postulated by theory. If a result addresses a problem raised by believing in some fields, symmetries or dimensions for which there is so far no observable evidence, then it is an internal result.

Thus, among the many published results, I have included here mostly substantial results. I have included internal results when they are important to judge the likelihood of the truth of the central conjectures of the theories.

Within the class of substantial questions we may make some distinctions according to subject. The focus of interest in this essay is on the questions that any quantum theory of gravity must answer. A second set of questions comes from cosmology. They arise from the existence of puzzles arising concerning cosmological data that appear to have no solution except in unknown effects at the Planck scale. While it is not necessary that a quantum theory of gravity answer these cosmological questions, there is still, because of this, a good possibility that a quantum gravity theory may make testable predictions about cosmological data. This is reason enough to include these questions in the present evaluation.

There are of course also questions about elementary particle physics. Here the two theories are in rather different situations. String theory claims to be a unified theory of all interactions, hence it must be judged on its ability to make verifiable predictions about the elementary particles. Loop quantum gravity makes no claim to be other than the quantum theory of gravity, and in fact appears able to incorporate equally well a wide variety of matter fields and interactions. So while loop quantum gravity can easily incorporate the standard model of particle physics, it, at least so far, makes no claims to explain any features of the standard model.

Here string theory has a big potential advantage. Given the fact that it is truly a unified theory, were it to make striking and unique predictions for elementary particle physics that were confirmed experimentally, this would be strong reason to believe in string theory. At

the same time, this is also a potential vulnerability, for if it makes no such predictions it loses credibility.

There is here a real difference between the two approaches. There is no *a priori* reason that the problem of quantum gravity is strongly linked to the question of unification. After all the quantum theory of electromagnetism, *QED*, has little to say about unification and does not strongly constrain the matter degrees of freedom or what other interactions there are in nature. At the most we can say that to eliminate a potential inconsistency at high energy-called the Landau ghost- *QED* should be imbedded in an asymptotically free gauge theory. But there are many of those, and even this does not imply the unification of all the gauge forces.

Nor is there an absolutely *compelling* reason to believe in a unification of gravity with the other forces. Gravity plays a unique role in physics, as it is connected with the geometry of space and time. Thus, it is only gravity that can be understood to be a consequence of the fact that the specification of inertial frames is local and dynamically determined. It is of course possible that, as has been proposed for decades, the other interactions also come from the dynamics of spacetime geometry, such as the curvature of extra dimensions. However, while this is an extremely attractive idea, it must also be admitted that there is so far no compelling argument from experiment or theory for either the existence of the extra dimensions or the necessity that the other forces be described in terms of them.

The best evidence that the problem of quantum gravity is related to the problem of unification comes instead from perturbation theory. It comes first from the fact that supersymmetry appears to be required to have a perturbative quantum theory that includes gravitons and is also exactly Lorentz invariant. Further, among the possible supersymmetric gravitational theories, we can make a good case for the likelihood of complete consistency only in the case of the string theories. This is consequential, and is a strong argument for taking string theory seriously, at the very least as an effective description of a fundamental theory, good at scales less than the Planck scale. But it could still be wrong, for example, it could be that Lorentz invariance is broken or modified at Planck scales[28]-[40]. Were this to be discovered experimentally (and, as we will mention below, there are experimental results that may be interpreted as indicating a failure of Lorentz invariance)[30] not only would string theory be not needed, but one of its main assumptions would have been falsified.

Finally, there are questions about foundational issues concerning quantum theory and the nature of time. The situation here is similar to that of unification. Good arguments have been put forward by several of the deepest thinkers in the field-people like Roger Penrose[50] and Gerard 't Hooft[51]-that the problem of quantum gravity cannot be solved without revising the principles of quantum theory. But there is no experimental evidence for such modifications and it remains possible that such arguments are wrong and that quantum gravity, like quantum electrodynamics, can be solved without forcing a deepening of our understanding of the principles of quantum theory.

One reason to side with the deep thinkers is the difficulty of formulating quantum theory sensibly in a cosmological context in which the observers must be part of the system[90, 64, 19]. But still, it may be that the problem of discovering the quantum theory of the

gravitational field in local regions of spacetime may be solved separately, while the problems of quantum cosmology remain open for smarter people in the future to finally resolve.

After listing the questions the theories may aspire to answer, I give a quick survey of the similarities and differences between the two theories. Indeed, it is striking and, I believe, non-trivial, that the two theories have a lot in common, so much so that any evaluation of their future must take into account the possibility that they will turn out to be different sides of a single theory. At the same time, there are big differences between them, and some of these can be recognized immediately. After this, we begin the detailed listing of results and open conjectures for each theory.

Before closing the introduction I should state my own situation with respect to the two theories. Since 1984 I have worked on both string theory and loop quantum gravity. If I have so far contributed more to loop quantum gravity, the majority of my papers since 1998 concern string or \mathcal{M} theory. I have also given graduate courses in both string theory and loop quantum gravity, I've had Ph.D. students and hired postdocs working in both areas and I attend conferences in both areas. So I think I do know both of them in enough technical detail to attempt this kind of evaluation. In particular, I have tried to make my own choices of which program to work on based on an objective evaluation of their potential to solve the key questions in quantum gravity. And, as the theories developed, I have made this choice differently at different times in the last 18 years.

Of course, I do not expect everyone will be happy with the conclusions I reach here. I myself was surprised by the conclusions I was led to by going through the exercise of writing this paper, and they have changed my own research priorities. But I do believe that any honest person who takes the time to acquaint themselves with the actual technical details of each theory sufficiently to understand the detailed statements of assumptions and results, will, if they reflect carefully on the actual evidence at hand, and if they are in fact open enough to accept any conclusion the evidence supports, reach essentially the same conclusions I do here.

As we proceed, I will lay out my conclusions with care, and with suitable attention to careful statements of assumptions and results. I am more than happy to discuss any of the conclusions I reach with anyone, and I am open to having my views changed, either by someone explaining something I missed or misunderstood, or, of course, by new results.

Other approaches

Before going on it is important to mention that string theory and loop quantum gravity are not the only approaches to quantum gravity that have been invented and studied. Other approaches include causal sets[52], dynamical triangulations[53], causal dynamical triangulations[54], twister theory[57], non-commutative geometry[58], supergravity, approaches based on analogies to condensed matter physics, etc. Each of these is motivated by rather compelling arguments, and each has been pursued vigorously by a community of smart people. Several of them, such as dynamical triangulations and causal dynamical triangulations, have achieved very significant results.

While none of these approaches has gained nearly the number of results found for string theory or loop quantum gravity, some of them do nevertheless address key issues and so deserve mention in any survey of progress in quantum gravity.

It is also the case that some aspects of some of these approaches have been incorporated into string theory or loop quantum gravity. For example, non-commutative geometry appears in both, and causal sets play a role in loop quantum gravity. To further complicate the situation, some approaches can, if one wishes, be considered to be subcases or limits of string theory or loop quantum gravity, but may also stand on their own. For example supergravity can be considered to be a limit of string theory, although a few purists may want to insist that there still may be a quantization of supergravity which is not a string theory. Similarly, dynamical triangulation models can be considered to comprise a class of loop quantum gravity models, and the methods used to study them likely extend to general loop quantum gravity models. But there is no necessity to consider them as loop quantum gravity models.

2 Physical questions the theories should answer

2.1 Questions concerning quantum gravity

We begin with the problems of quantum gravity itself. The correct quantum theory of gravity must:

1. *Tell us whether the principles of general relativity and quantum mechanics are true as they stand, or are in need of modification.*
2. *Give a precise description of nature at all scales, including the Planck scale.*
3. *Tell us what time and space are, in language fully compatible with both quantum theory and the fact that the geometry of spacetime is dynamical. Tell us how light cones, causal structure, the metric, etc are to be described quantum mechanically, and at the Planck scale.*
4. *Give a derivation of the black hole entropy and temperature. Explain how the black hole entropy can be understood as a statistical entropy, gotten by coarse graining the quantum description.*
5. *Be compatible with the apparently observed positive, but small, value of the cosmological constant. Explain the entropy of the cosmological horizon.*
6. *Explain what happens at singularities of classical general relativity.*
7. *Be fully background independent. This means that no classical fields, or solutions to the classical field equations appear in the theory in any capacity, except as approximations to quantum states and histories.*

8. *Predict new physical phenomena, at least some of which are testable in current or near future experiments.*
9. *Explain how classical general relativity emerges in an appropriate low energy limit from the physics of the Planck scale.*
10. *Predict whether the observed global lorentz invariance of flat spacetime is realized exactly in nature, up to infinite boost parameter, or whether there are modifications of the realization of lorentz invariance for Planck scale energy and momenta.*
11. *Provide precise predictions for the scattering of gravitons, with each other and with other quanta, to all orders in a perturbative expansion around the semiclassical approximation.*

These are a lot of questions, but it is hard to imagine believing in a quantum theory of space and time that did not answer each one. However, there is one that cannot be over-emphasized, which is the requirement of background independence. There are two reasons for making this requirement. The first is a matter of principle. Over the whole history of physics, from the Greeks onwards, there have been two competing views about the nature of space and time. The first is that they are not part of the dynamical system, but are instead eternally fixed, non-dynamical aspects of the background, against which the laws of nature are defined. This was the point of view of Newton and it is generally called the *absolute* point of view. The second view holds that the geometry of space and time are aspects of the dynamical system that makes up the universe. They are then not fixed, but evolve as does everything else, according to law. Further, according to this view, space and time are *relational*. This means there is no absolute meaning to where or when an event occurs, except as so far as can be determined by observable correlations or relations with other events. This was the point of view of Leibniz, Mach and Einstein and is called the *relational* point of view.

Einstein's theory of general relativity is an instantiation of the relational point of view. The observations that show that gravitational radiation carries energy away from binary pulsars in two degrees of freedom of radiation, exactly as predicted by Einstein's theory, may be considered the experimental death blow to the *absolute* point of view. The fact that two, and not five, degrees of freedom are observed means that the gauge invariance of the laws of nature includes spacetime diffeomorphism invariance. This means that the metric is a completely dynamical entity, and no component of the metric is fixed and non-dynamical.

As argued by Einstein and many others since, the diffeomorphism invariance is tied directly to the background independence of the theory. This is shown by the hole argument[59], and by Dirac's analysis of the meaning of gauge symmetry[60]. There are good discussions of this by Stachel[61], Barbour[62], Rovelli[63] and others[19, 64].

Thus, classical general relativity is background independent. The arena for its dynamics is no spacetime, instead the arena is the configuration space of all the degrees of freedom of the gravitational field, which is the metric modulo diffeomorphisms.

Now we can ask, must the quantum theory of gravity also be background independent? To have it otherwise would be as if some particular classical Yang-Mills field was required to *define* the quantum dynamics of *QCD*, while no such fixed, non-dynamical field need be specified to define the classical theory. Still a number of people have expressed the view that perhaps the quantum theory of gravity requires a fixed non-dynamical spacetime background for its very definition. This seems almost absurd, for it would mean taking some particular solution (out of infinitely many) to the classical theory, and making it play a preferred role in the quantum theory. Moreover, there must be no experimental way to discover which classical background was taken to play this preferred role, for if any effect which depended on the fixed background survived in the low energy limit, it would break diffeomorphism invariance. But this would in turn mean that diffeomorphism invariance was not an exact gauge symmetry in the low energy limit, and this would imply that more than two degrees of freedom of the metric would be excited when matter accelerated. But this would contradict the extreme sensitivity of the agreement between general relativity and the rate of decay of binary pulsar orbits.

Thus, arguments from both principle and from experiment reinforce the conclusion that nature is constructed in such a way that, even in the quantum domain, all the degrees of freedom of the spacetime geometry are dynamical. But if this is the case no fixed classical metric can play any role in the formulation of the quantum theory of gravity⁴.

2.2 Questions concerning cosmology

Next we mention cosmological puzzles that are so far unsolved and that are widely believed to require Planck scale physics for their resolution.

1. *Explain why our universe apparently began with extremely improbable initial conditions[50].*
2. *In particular, explain why the universe had at grand unified times initial conditions suitable for inflation to occur or, alternatively, give an alternate mechanism for inflation or a mechanism by which the successes of inflationary cosmology are duplicated.*
3. *Explain whether the big bang was the first moment of time, or whether there was something before that.*

⁴It is sometimes argued in rebuttal that an acceptable theory may be formulated in such a way that the quantum theory depends on a classical background, but any of a large number of backgrounds may be used, so that the theory does not require one special background. This misses the point, as such a theory in fact consists of a long list of quantum theories, one for each background. This fails to realize the idea that quantum spacetime as a whole is dynamical, so that the different backgrounds arise as solutions of the quantum dynamics. It is not enough that the different backgrounds may be solutions of different *classical* equations, for that leads to a mixed and most likely inconsistent theory in which the geometry is split in such a way that one part (the background) solves a classical equation, while the other part (the gravitational waves “on the background” satisfy quantum equations that depend on the choice of background. Such an approach may arise as an approximation to a fundamental theory, but it cannot be a fundamental theory in itself.

4. *Explain what the dark matter is. Explain what the dark energy is. Explain why at present the dark matter is six times as dense as ordinary hadronic matter, while the dark energy is in turn twice as dense as the dark matter.*
5. *Provide predictions that go beyond those of the currently standard model of cosmology, such as corrections to the CMB spectra predicted by inflationary models.*

2.3 Questions concerning unification of the forces

Next, we mention problems in elementary particle physics that must be resolved by any unified theory of all the interactions. As string theory must, if true, be such a theory, it must be evaluated against progress in answering these questions. It is also possible, but not as necessary, that loop quantum gravity offer answers to some of these questions.

1. *Discover whether there is a further unification among the forces, including gravity or not.*
2. *Explain the general features of the standard model of elementary particle physics. i.e. explain why the forces are described by a spontaneously broken gauge theory with group $SU(3) \times SU(2) \times U(1)$, with fermions in the particular chiral representations observed.*
3. *Explain why there is observed a large hierarchy in the ratio of masses observed, from the Planck mass, down to the neutrino masses and finally down to the cosmological constant. Discover the mechanism by which the hierarchy was created, whether by spontaneous breaking of a more unified theory or by other means. Explain why the cosmological constant is so small in Planck units.*
4. *Explain the actual values of the parameters of the standard model: masses, coupling constants, mixing angles etc. Explain the observed value of the cosmological constant.*
5. *Tell us whether there is a unique consistent theory of nature that implies unique predictions for all experiments or whether, as has been sometimes proposed, some or all of the questions left open by the standard model of particle physics are to be answered in terms of choices among possible consistent phenomenologies allowed by the fundamental theory.*
6. *Makes some experimental predictions for phenomena that are unique to that theory and which are testable in present or near future experiments?*

2.4 Foundational questions

Finally, there are the questions in the foundations of quantum theory, which many people believe are closely related to the problem of quantum gravity.

1. *Resolve the problem of time in quantum cosmology*

2. *Explain how quantum mechanics is to be modified to apply to a closed system such as the universe that contains its own observers*

3 A tale of two theories

Before listing the main results and open issues of each theory, it is useful to survey the main points in common, and main differences of the two theories. Both the similarities and differences are striking and non-trivial, and it is probably useful for the reader if they are highlighted here, before we are involved, necessarily, in a great deal of details and fine distinctions necessary to reach a careful evaluation of each theory.

3.1 Common postulates

String theory and loop quantum gravity are each a development of a set of ideas originally introduced in the 1960's to understand hadronic physics. As such they share some common postulates.

- **The fundamental theory is not a conventional Poincare invariant local field theory.**
- **The fundamental excitations are extended objects.** These include one dimensional excitations and two (and perhaps higher dimensional) membrane-like excitations.
- **Duality** The one dimensional excitations have a dual description as quanta of electric flux of a non-abelian gauge theory. The higher dimensional excitations have a dual description in terms of higher dimensional electric and magnetic fluxes.
- **The holographic principle.** This is a recently proposed principle which, if true, is the first principle we have that uniquely concerns the quantum theory of gravity. It says, roughly, that observables for quantum gravity theories in d spacetime dimensions can be evaluated in terms of data on $d - 1$ dimensional surfaces[68]-[72]⁵. These surfaces may be boundaries of the spacetime or, in the cosmological case, may be surfaces embedded in the spacetime.

Different versions of the holographic principle have been proposed, which differ on the extent to which the theory can be completely reduced to a dynamical theory on the lower dimensional surface. For more details see[72].

⁵The principle was independently proposed by Louis Crane[68] and Gerard 't Hooft[69] in 1993. It was applied to string theory by Susskind in [70] where it inspired developments such as the AdS/CFT conjecture. Crane's formulation inspired key developments in loop quantum gravity including the discovery that Chern-Simons theory plays a role in describing the quantum geometry of boundaries and horizons, leading to the present understanding of black hole entropy[98, 100].

The fact that string theory and loop quantum gravity share these common postulates is reflected in the fact that the mathematics employed in their formulation overlaps. For example, they both employ conformal field theory and the representation theory of quantum groups. Both theories may be formulated in a language in which all the degrees of freedom are represented as large matrices. These formulations are non-perturbative in the sense that the dynamics of the matrices code an infinite number of terms in a perturbation theory⁶.

However there are very significant differences as well.

3.2 Why string theory and loop quantum gravity differ

String theory and loop quantum gravity both begin by taking the one dimensional extended objects, which by duality correspond to electric flux lines of a quantized gauge field, to be fundamental degrees of freedom of the theory. They differ in two ways:

- **Difference one.** *The strings are taken to move in a classical background characterized by a fixed choice of a metric and other classical fields. The loops are taken to exist at a more fundamental level, at which there are no classical metrics or other fields.*
- **Difference two.** *The gauge field in the case of loops is taken to gauge all or part of the local lorentz transformations. The gauge field in the case of open strings is taken to correspond to a Yang-Mills field.*
- **Difference three.** *The two approaches take very different strategies to address the failure of general relativity to exist as a perturbatively renormalizable quantum field theory. These have to do with the attitude to the physical assumptions that underlie the use of perturbation theory. These postulates include i) spacetime is smooth down to arbitrarily small scales, so that there are linearized perturbations of arbitrarily short wavelength. ii) Global lorentz symmetry is an exact symmetry of the spectrum of fluctuations around the quantum state corresponding to Minkowski spacetime, good to arbitrarily small wavelength and large boost parameter.*

These two postulates are assumed by string theory to be exact. The attitude taken is to search for a perturbative theory incorporating gravitons in which they can be exactly realized.

In contrast, loop quantum gravity takes the attitude that we must make a quantization of general relativity that does not rely on these two assumptions. Indeed, as global lorentz invariance is not a symmetry of classical general relativity, it cannot be assumed in any exact quantization of the theory. These two assumptions are then to be tested, in the sense that one must see to what extent they are recovered in the classical limit of the quantum theory. In fact, as we shall see, the evidence is that they are false in at least one consistent quantization of general relativity.

⁶For the matrix formulation of loop quantum gravity see[65], for string and \mathcal{M} theory formulated as a matrix model see [66, 67]

As a result of these differences the two theories have different postulates. They lead, as well, to very different physical pictures. Consequently, the two theories make quite different predictions for future experiments. It is worth mentioning these at the beginning.

3.3 Characteristic predictions of string theory

String theory appears to require that the world have large numbers of so far unobserved dimensions, degrees of freedom and symmetries[4, 5, 6]. While we will discuss this in detail below, it can be said that string theory requires that nature have 6 or 7 dimensions of space beyond the three that are observed. It also predicts the existence of a new kind of symmetry, called supersymmetry, which is also so far unobserved. This is a symmetry that relates fermions to bosons. Unfortunately, it appears that supersymmetry cannot be used to relate any of the presently known fermions to any of the presently known bosons. Thus, supersymmetry, and string theory, predict that there are a great many unobserved elementary particles.

Two things must then be said. There is so far no evidence at all from observation for any of the additional dimensions, symmetries or particles that string theory predicts⁷. Second, string theory is not unique in predicting any of these features. String theory was preceded by the study of higher dimensional theories and ordinary theories with supersymmetry. These theories continue to be studied independently of string theory. It is not easy to point to a doable experiment that would confirm a prediction of string theory, uniquely, that is not also a prediction of an ordinary supersymmetric or higher dimensional field theory.

There is one assumption that string theory makes which is subject to experimental test. This is that special relativity holds, at all distance scales, in its original form given by Einstein. In technical language this means that the theory assumes that lorentz invariance is an exact symmetry of the world we live in, neglecting only effects due to the curvature of spacetime.

3.4 Characteristic predictions of loop quantum gravity.

Loop quantum gravity also leads to characteristic predictions of new phenomena, but of a rather different type. In fact loop quantum gravity is completely compatible with the postulate that the world has only three spatial dimensions, and one time dimension, and is known to be compatible with a large range of assumptions about the matter content of the world, including the standard model. So it does not require any dimensions, symmetries or degrees of freedom beyond what are observed. At the same time, there are versions of loop

⁷There are some facts that are taken as possible indirect evidence for supersymmetry in particle physics. One has to do with the question of whether the gauge and Yukawa coupling constants meet at a single grand unification scale. There is approximate but not exact unification in the standard model. The unification is closer in the minimal supersymmetric standard model, in that the triangle made by the three running coupling constants is smaller and it is more plausible that unification is achieved by threshold effects[205]. However, the running of the coupling constants may also be influenced by other factors such as neutrino masses[206].

quantum gravity that incorporate supersymmetry (at least up to $N = 2$,) and many results extend to higher dimensions. So were experimental evidence for either supersymmetry or higher dimensions this would not pose a problem for loop quantum gravity.

Instead, the predictions of loop quantum gravity concern the structure of space and spacetime at very short distances. In particular, loop quantum gravity predicts that the smooth picture of spacetime in classical general relativity is actually only a coarse grained approximation to a discrete structure, in which surfaces and regions can have only certain, discrete quantized values of areas and volume[10, 18, 73, 74, 17]. Loop quantum gravity makes specific predictions for the discrete quantum geometry at short distances. Furthermore, these predictions are derived from first principles, hence they are not adjustable. In this way loop quantum gravity is different from previous approaches which postulate some form of discrete structure as a starting point, rather than deriving it as a consequence of the union of quantum theory and general relativity.

It turns out that this has consequences for the question of whether special relativity, and lorentz invariance, is exactly true in nature, or is only an approximation which holds on scales much longer than the Planck scale[28]-[40]. Several recent calculations, done with different methods[36]-[38], yield predictions for modifications to the energy momentum relations for elementary particles. These are of the form,

$$E^2 = p^2 + M^2 + \alpha l_{Pl} E^3 + \beta l_{Pl}^2 E^4 + \dots \quad (1)$$

where predictions have been found for the leading coefficients α , which generally depend on spin and helicity[36]-[38].

This is then an area of disagreement with string theory. Further, these modifications appear to be testable with planned experiments[28, 30, 39, 40]. Hence the different predictions of string theory and loop quantum gravity concerning the fate of lorentz invariance offer a possibility of experimentally distinguishing the theories in the near future.

4 The near term experimental situation

The most important development of the last few years in quantum gravity is the realization that it is now possible to probe Planck scale physics experimentally. Depending on dynamical assumptions there is now good experimental sensitivity to the α terms in (1) for photons, electrons and protons. Increased sensitivity is expected over the next few years from a number of other experiments so that it is not impossible that even if the leading order E^3 terms are absent, it will be possible to put order unity bounds on β , the coefficient of the E^4 term.

However it is crucial to mention that to measure α and β one has to specify how lorentz invariance is treated in the theory. There are two very different possibilities which must be distinguished.

- **Scenario A)** The relativity of inertial frames is broken and there exists a preferred frame. In this case the analysis has to be done in that preferred frame. The most

likely assumption is that the preferred frame coincides with the rest frame of the cosmic microwave background. In such theories energy and momentum conservation are assumed to remain linear.

- **Scenario B)** The relativity of inertial frames is preserved, but the lorentz transformations are realized non-linearly when acting on the energy and momentum eigenstates of the theory. Such theories are called modified special relativity or doubly special relativity. Examples are given by some forms of non-commutative geometry, for example, κ -Minkowski spacetime[32]. In all such theories energy and momentum conservation become non-linear which, of course, effects the analysis of the experiments. In some, but not all, cases of such theories, the geometry of spacetime becomes non-commutative.

Among the experiments which either already give sufficient sensitivity to measure α and β , or are expected to by 2010 are,

1. There are apparent violations of the GZK bound observed in ultra high energy cosmic rays (UHECR) detected by the AGASA experiment[29]. The experimental situation is inconsistent, but the new AUGER cosmic ray detector, which is now operational, is expected to resolve the situation over the next year or two. If there is a violation of the GZK bound, a possible explanation is Planck scale physics coming from (1) [30].
In Scenario A) violations of the GZK bounds can be explained by either E^3 or E^4 terms in the proton energy-momentum relation. However, in case B) it is less natural to explain a violation of the GZK bounds by means of a Planck scale modification of the energy-momentum relations, but there are proposals for forms of such theories that do achieve this.
2. A similar anomaly is possibly indicated in Tev photons coming from blazars[41]. Similar remarks apply as to the explanatory power of Scenarios A) and B) in the event that the anomaly exists.
3. A consequence of (1) is an energy dependent speed of light. This effect can be looked for in timing data of gamma ray busts. Present data bounds $\alpha < \approx 10^3$ and data expected from the GLAST experiment is expected to be sensitive to α of order one in 2006 [42]. Note that this applies to both Scenarios A) and B).
4. Present observations of synchrotron radiation in the Crab nebula, together with reasonable astrophysical assumptions, put *very strong* (of order 10^{-9} !!) bounds on α for photons and electrons, in the case of Scenario A only[43].
5. Present data from precision nuclear and atomic physics experiments puts very tight bounds on α for photons, electrons and hadrons, again in Scenario A), only[44].
6. Present data from the absence of vacuum cherenkov effects puts interesting bounds on α in the case of Scenario A) [39].

7. Observations of birefringence effects in polarized light from distant galaxies puts tight bounds on a possible helicity dependent α [45].
8. Observations of phase coherence in stellar and galactic interferometry is expected, given certain assumptions, to put order one bounds on α in the near future [46].
9. Certain hypotheses about the Planck scale lead to the prediction of noise in gravitational wave detectors that may be observable at LIGO and VIRGO[47].
10. Under some cosmological scenarios, modifications of the form of (1) lead to distortions of the CMB spectrum that may be observable in near future observations[48].

We may summarize this situation by saying that a theory of quantum gravity that leads to Scenario A) and predicts an energy momentum relation (1) with α order unity is plausibly already ruled out. This is shocking, as it was commonly said just a few years ago that it would be impossible to test any physical hypotheses concerning the Planck scale.

We can also mention three other kinds of experiments that by 2010 will have relevance for the problem of quantum gravity

1. Evidence for or against supersymmetry may be detected at the Tev scale in accelerators.
2. The equation of state of the dark energy will be measured in near future experiments. Some proposals for dark energy are based on modifications of energy momentum relations of the form of (1).
3. There are observations that appear to indicate that the fine structure constant is time dependent[49]. These will be confirmed or go away. If the claim is substantiated this offers a big challenge to the effective field theory understanding of low energy physics.

The combination of all these experimental possibilities signals that the long period when fundamental physics developed independently of experiment is soon coming to a close. As indicated above, the possible experimental outcomes may rule out either string theory or loop quantum gravity by 2010. Certain hypotheses about Planck scale physics, which lead to preferred frame effects of scenario A) are already ruled out or tightly constrained by observation.

5 Postulates and main results of loop quantum gravity

For precision it turns out to be necessary to distinguish two forms of loop quantum gravity, which I will call versions I and II.

5.1 Postulates of loop quantum gravity I

What I will mean by loop quantum gravity, *version I* is the theory which is the quantization of the Einstein's equations, coupled to arbitrary matter fields, in 3 + 1 dimensions.

- *The quantum theory of gravity is the quantization of general relativity, or some extension of it, involving matter fields, such as supergravity. The quantization is done using standard non-perturbative Hamiltonian and path integral methods, applied to the phase space coordinatized in terms of an alternative set of variables. The configuration variables are taken to be components of the spacetime connection, so that general relativity in a certain precise sense is expressed in terms of a gauge theory⁸.*
- *The quantization must be done in a manner that preserves the background independence of classical general relativity, and hence exactly realizes diffeomorphism invariance.*

In loop quantum gravity I the only non-dynamical structure that is fixed is a three manifold Σ , with a given topology and differential structure. There are no classical fields such as metrics, connections or matter fields on Σ . The only exception is in modeling the quantization of spacetime regions with boundary, as in the asymptotically flat or *AdS* context, or in the presence of a black hole or cosmological horizon. In these cases fields may be fixed on the boundary $\partial\Sigma$ to represent physical conditions held fixed there.

5.2 The main results of loop quantum gravity I

1. The states of the theory are known precisely. The Hilbert space \mathcal{H}^{diffeo} of spatially diffeomorphism invariant states of general relativity in 3+1 dimensions has an orthonormal basis, whose elements are in one to one correspondence with the diffeomorphism equivalence classes of embeddings of certain labeled graphs, called spin networks[79], into Σ [17].

A labeled graph is a graph whose edges and vertices have attached to them elements of a certain set of labels. In the case of pure general relativity with vanishing cosmological constant, the labels on the edges are given by ordinary $SU(2)$ spins. There are also labels on nodes of the spin networks, which are invariants or intertwiners from the representation theory of $SU(2)$. For details see [79, 17]..

2. Certain spatially diffeomorphism invariant observables have been constructed. After a suitable regularization procedure these turn out to be represented by *finite* operators

⁸These variables, and the simplifications they bring about were discovered by Sen[12] and formalized by Ashtekar[13] in the Hamiltonian formalism and by Plebanski[76] and others[77, 78] in the lagrangian formalism. By now several different connections are used in loop quantum gravity. These include the self-dual part of the spacetime connection[12, 13], and a real $SU(2)$ connection introduced by Barbero[123] and exploited by Thiemann[97]. There are also alternate formulations that use both the left and right handed parts of the spacetime connection, [101, 23].

on $\mathcal{H}^{diff eo}$, the space of spin network states[10, 18, 74]. These include the volume of the universe, the area of the boundary of the universe, or of any surface defined by the values of matter fields. Other operators also have been constructed, for example an operator that measures angles in the quantum geometry[75]. These operators all preserve the diffeomorphism invariance of the states[84].

3. The area and volume operators have discrete, finite spectra, valued in terms of the Planck length[18, 74]. There is hence a smallest possible volume and a smallest possible area, of order of the Planck volume and area. The spectra have been computed in closed form.
4. The area and volume operators can be promoted to genuine physical observables, by gauge fixing the time gauge so that at least locally time is measured by a physical field[83, 84]. The discrete spectra remain for such physical observables, hence the spectra of area and volume constitute genuine physical predictions of the quantum theory of gravity.
5. Due to the existence of minimal physical volumes and areas, the theory has no excitations that correspond to gravitons or matter degrees of freedom with wavelengths shorter than the Planck length[114, 122].
6. Among the operators that have been constructed and found to be finite on $\mathcal{H}^{diff eo}$ is the Hamiltonian constraint (or, as it is often called, the *Wheeler de Witt* equation[80]-[82].) Not only can the Wheeler deWitt equation then be precisely defined, it can be solved exactly. Several infinite sets of solutions have been constructed, as certain superpositions of the spin network basis states, for all values of the cosmological constant[15, 97]. These are exact, physical states of quantum general relativity.

If one fixes a physical time coordinate, in terms of the values of some physical fields, one can also define the Hamiltonian for evolution in that physical time coordinate[83] and it is also given by a finite operator on a suitable extension of $\mathcal{H}^{diff eo}$ including matter fields.

7. The dynamics of the spin network states can be expressed also in a path integral formalism, called spin foams[85]-[96]. The histories by which spin network states evolve to other spin network states, called spin foam histories, are explicitly known. A spin foam history is a labeled combinatorial structures, which can be described as a branched labeled two complex. Spin foam models have been derived in several different ways, and the results agree as to the general form of a spin foam amplitude. These include: 1) by exponentiation of the Hamiltonian constraint, 2) directly from a discrete approximation to the classical spacetime theory, 3) by constraining the summations in a finite state sum formulation of a four dimensional topological invariant, 4) from a matrix model on the space of fields over the group, 5) by postulating spacetime events are local moves in spin networks.

Evolution amplitudes corresponding to the quantization of the Einstein equations in $3 + 1$ dimensions, are known precisely[11] for vanishing and non-vanishing values of the cosmological constant, and for both the Euclidean and Lorentzian theories.

The sum over spin foams has two parts, a sum over graphs representing histories of spin networks, and, on each, a sum over the labels. The sums over labels are known from both analytic and numerical results to be convergent[127, 128] for some spin foam models, including some corresponding to the quantization of the Einstein equations in $2 + 1$ and $3 + 1$ dimensions.

For some spin foam model in $2 + 1$ dimensions, it has been shown that the sum over spin foam histories is Borel summable[186].

The physical inner product, which is the inner product on solutions to all the constraints, has an exact expression, given in terms of spin foam models[86].

8. All of the forgoing results have been extended to quantum general relativity with the standard matter fields, including Yang-Mills fields, spinors, scalars and Kalb-Ramond fields.
9. Spin foam models appropriate for Lorentzian quantum gravity, called causal spin foams, have quantum analogues of all the basic features of general relativistic spacetimes⁹. These include dynamically generated causal structure, light cones and a discrete analogue of multifingered time, which is the freedom to slice the spacetime many different ways into sequences of spatial slices[89]. The spatial slices are spin networks, which are quantum analogues of spatial geometries.
10. Several kinds of boundaries may be incorporated in the theory including timelike boundaries, in the presence of both positive and negative cosmological constant, and null boundaries such as black hole and cosmological horizons[98]-[101]. In all these cases the boundary states and observables are understood in terms of structures derived from Chern-Simons theory.
11. The boundary Hilbert spaces decompose into eigenspaces, one for each eigenvalue of the operator that measures the area of the boundary[98]. For each area eigenvalue, the Hilbert space is finite dimensional. The entropy may be computed and it agrees precisely with the Beckenstein-Hawking semiclassical result,

$$S = \frac{A[S]}{4\hbar G_{Newton}} \quad (2)$$

Among the boundaries that can be studied are horizons. The boundary theory then provides a detailed microscopic description of the physics at the boundary. Furthermore, the prediction of Bekenstein and Hawking that an horizon should have the

⁹For more details on these models and the resulting physical picture, see[90].

entropy (2) is completely explained in terms of the statistical mechanics of the state spaces associated with the degrees of freedom on the horizon. This has been found to work for a large class of black holes, including Schwarzschild black holes[99, 100].

The calculation of the entropy involves a parameter, which is called the Imirzi parameter. This can be understood either as a free parameter that labels a one dimensional family of spin network representations, or as the (finite) ratio of the bare to renormalized Newton's constant. The Imirzi parameter is fixed precisely by an argument invented by Dreyer, involving quasi normal modes of black holes[190]. Dreyer's argument depends on a remarkable precise coincidence between an asymptotic value of the quasi normal mode frequency and a number which appears in the loop quantum gravity description of horizons. The value of the asymptotic quasi normal mode frequency was at first known only numerically, but it has been very recently derived analytically by Motl[191]. Once Dreyer's argument fixes the Imirzi parameter, the Bekenstein-Hawking relation (2) is predicted exactly for all black hole and cosmological horizons¹⁰.

12. Corrections to the Bekenstein entropy have been calculated and found to be logarithmic[103].
13. Suitable approximate calculations reproduce the Hawking predict a discrete fine structure in the Hawking spectrum[104, 105]. At the same time, the spectrum fills in and becomes continuous in the limit of infinite black hole mass. This fine structure stands as another definitive physical prediction of the theory.

Thus, to summarize, loop quantum gravity leads to a detailed microscopic picture of the quantum geometry of a black hole or cosmological horizon[100]. This picture reproduces completely and explains the results on the thermodynamic and quantum properties of horizons from the work of Bekenstein[1], Hawking[2] and Unruh[3]. This picture is completely general and applies to all black hole and cosmological horizons.

14. For the case of non-vanishing cosmological constant, of either sign, there is an exact physical state, called the Kodama state, which is an exact solution to all of the quantum constraint equations, whose semiclassical limit exists[108]. That limit describes deSitter or anti-deSitter spacetime. Solutions obtained by perturbing around this state, in both gravitational[38] and matter fields[109], reproduce, at long wavelength, quantum field theory in curved spacetime and the quantum theory of long wave length, free gravitational waves on deSitter or anti-deSitter spacetime.
15. The inverse cosmological constant turns out to be quantized in integral units, so that $k = 6\pi/G\Lambda$ is an integer[98].
16. The thermal nature of quantum field theory in a deSitter spacetime is explained in terms of a periodicity in the configuration space of the exact quantum theory of general relativity[38].

¹⁰Dreyer's calculation leads also to the conclusion that the "punctures" where spin networks meet the horizon are dominated by spin 1 edges.

17. A large class of states are known which have course grained descriptions which reproduce the geometry of flat space, or any slowly varying metric[114, 115]. Linearizing the quantum theory around these states yields linearized quantum gravity, for gravitons with wavelength long compared to the Planck length[116]. It is also understood how to construct coherent states which are peaked around classical configurations[129].
18. A reduction of the exact physical state space to states which are spatially homogeneous is known, and the reduction of the dynamics to this subspace of states is known[117]. (This is different from the usual quantum cosmology in that the reduction to homogeneous states is done in the Hilbert space of the full theory, rather than before quantization.) The evolution of these states has been studied in detail and it has been found generically that the usual FRW cosmology is reproduced when the universe is very large in Planck units. At the same time the cosmological singularities are removed, and replaced by bounces where the universe re-expands (or pre-contracts). When couplings to a scalar field are included, there is a natural mechanism which generates Planck scale inflation as well as a graceful exit from it[117].
19. Many of these results extend to quantum supergravity for $N = 1$ and several have been studied also for $N = 2$ [24].
20. The same methods can also be used to solve quantum gravity in $2 + 1$ dimensions[118] and in many $1 + 1$ dimensional reductions of the theory[119]. They also work to solve a large class of topological field theories[106, 107], giving results equivalent to those achieved by other methods. Further, loop methods applied to lattice gauge theories yields results equivalent to those achieved by other methods[120].
21. In both flat space and around deSitter spacetime, one may extend the calculations that reproduce quantum theory for long wavelength gravitons and matter fields to higher energies. These calculations reveal the presence of corrections to the energy-momentum relations of the form of (1). However, now the parameters α and β are computable constants, that depend on the ground state wavefunctional[36, 37, 38]. These represent further predictions of the theory.
22. Many of these results have been checked by being derived by several different methods, involving different regularization procedures. Some of these employ a high energy physics level of rigor, while other methods are fully rigorous, at the level of mathematical quantum field theory[121, 97, 122]. All the key results have been verified by being rederived with completely rigorous methods.

On the basis of these results, it can be claimed that loop quantum gravity I is both the correct quantization of general relativity and a physically plausible candidate for the quantum theory of gravity. It appears to provide a precise answer to the first 9 questions in my list.

The failure of quantum general relativity in perturbation theory is explained by the fact that there are, in this quantization of general relativity, no degrees of freedom that correspond to gravitons or other perturbative quanta with wavelength shorter than the Planck scale. The ultraviolet divergences are eliminated because a correct quantization, that exactly realizes spatial diffeomorphism invariance, turns out to impose an ultraviolet cutoff on the physical spectrum of the theory. The assumptions mentioned above, that spacetime is smooth and lorentz invariant at arbitrarily short scales, are not used in the quantization procedure, and in fact turns out to be contradicted by the results.

Readers with a training in perturbative quantum field theory may be skeptical of these claims. There are two important things that may be said in response. First the results are not about generic perturbatively non-renormalizable theories. The key results of both the hamiltonian and path integral quantizations follow from two necessary features special to gravitational theories¹¹. The first is the spatial diffeomorphism invariance. This imposes a method of quantization that would fail for ordinary Poincare invariant quantum field theories. This is not based on Fock space, it is based on a certain representation of the algebra of Wilson loop observables, which allows a rigorous[121, 122] formulation of the theory incorporating an exact unitary representation of the group of spatial diffeomorphisms.. As a result, many potential divergences are eliminated by the requirement that operators be constructed by regularization procedures that preserve the diffeomorphism invariance of the states in the limit the regulator is removed.

The second feature is that the actions for many known gravitational theories can be put in the form which is closely related to a class of topological field theories[38, 26]. These are called BF theories because their actions are of the form of $\int Tr B \wedge F$. The actions of these gravitational theories are the sum of this term with a constraint, non-derivative and quadratic in B . Theories which can be so expressed may be called *constrained topological field theories*. These include general relativity in all dimensions[26] and supergravity, at least in $d = 4$ for $N = 1, 2$ and in $d = 11$ [27].

The combination of these two features makes possible the unexpected results cited.

It should also be said that all of the key results in the hamiltonian theory, and some in the path integral theory, are understood completely rigorously[122, 121]. A reader may doubt that the world is constructed as the quantization of general relativity, but it is no longer an option to disagree that these methods lead to a rigorously understood class of diffeomorphism invariant quantum field theories in four dimensions. Given the non-triviality of the existence of a class of quantum field theories that implement exact diffeomorphism invariance while still having local degrees of freedom, it is hard to believe that there are not important things to learn from them about how nature succeeds in unifying the postulates of quantum theory with the basic postulates of general relativity.

There is in fact not one point on which the theory has failed, and when a few times, there has appeared an obstacle, it has been removed by further research. For example,

¹¹These are described in detail in the cited references. It is fair to say that any criticism of these results is ill-informed if it is not based on a technical understanding of how these two features are implemented in the quantization procedure.

at one point a problem appeared with the action of the Hamiltonian constraint, which suggested difficulties with the recovery of general relativity in the low energy limit[112, 113]. This problem was addressed and solved, and in more than one way: first by the spin foam formalism, which showed that certain missing amplitudes in the Hamiltonian picture are there in the path integral formalism, and do resolve the problem[86, 89]. Then it was resolved a second time when it was found that for any nonvanishing value of the cosmological constant the missing amplitudes are automatically incorporated[92].

Similarly, while the problem of the recovery of general relativity in the low energy limit of the theory is still unsolved for zero cosmological constant, there is a strong indication that the existence of the Kodama state allows a satisfactory solution of the problem so long as the bare cosmological constant is non-zero[108, 109, 110, 38].

5.3 Loop quantum gravity II

While loop quantum gravity I so far appears to be satisfactory as both a quantization of general relativity and a quantum theory of gravity, it may very well be that the quantization of general relativity does not in fact describe nature. The dimension of spacetime, physical degrees of freedom, and fundamental symmetries may be different from those which are presently observed. It turns out that there is a natural class of models which generalizes loop quantum gravity which addresses these possibilities. These may be called *loop quantum gravity II* models¹².

To discuss these we may observe that the mathematical language of states, histories, boundaries and observables which is derived in the case of quantum general relativity can be easily generalized to give a large class of fully background independent quantum theories of spacetime. To describe the kinematics of a theory of this kind one must specify only an algebra (or superalgebra) whose representation theory is used to label the spin networks. The graphs on which the spin networks are based are defined combinatorially, so that the need to specify the topology and dimension of the spatial manifold is eliminated[89, 92]. In such a theory the dimension and topology are dynamical, and different states may exist whose coarse grained descriptions reveal manifolds of different dimensions and topology.

The main postulate of loop quantum gravity II may be stated as follows:

- The states of a quantum theory of gravity are given by abstract spin networks associated with the representation theory of a given Hopf algebra or superalgebra, \mathcal{A} ¹³.
- The histories of the theory are given by spin foams labeled by the same representations. The dynamics of the theory is specified by evolution amplitudes assigned to the nodes of the spin foams (or equivalently to local moves by which the spin networks evolve).

¹²Another name for these theories which is sometimes used is *categorical state sum models*, because they may be formulated elegantly in the language of tensor categories[68].

¹³Here a spin network is a graph whose edges are labeled by representations of \mathcal{A} and whose nodes are represented by invariants of \mathcal{A} .

Many of the results of loop quantum gravity I apply in a suitably generalized form to loop quantum gravity II. Loop quantum gravity II thus specifies a large class of background independent quantum theories of space, time and gravitation. There are even proposals that a particular form of loop quantum gravity II may be the background independent form of string theory[174].

There are many loop quantum gravity II models that are not loop quantum gravity I. Examples include dynamical triangulation models[53] and causal dynamical triangulation models[54]-[56]. These take the trivial case in which the algebra \mathcal{A} contains only the identity operator, but they have states which are described in terms of graphs and histories that satisfy the definition of a spin foam model. We will discuss the results achieved with these models below.

Finally, it should be mentioned that, at least in 2+1 and 3+1 dimensions the cosmological constant is coded in a natural way in all loop quantum gravity theories, which is that it is related to the quantum deformation of the algebra of representations of the local lorentz group[98, 124].

One may think of loop quantum gravity II theories in the following terms. Suppose we want to construct a completely background independent quantum field theory. Such a theory must be independent of any of the ingredients of a classical field theory, including manifolds, coordinates, metrics, connections and fields. What is left of quantum theory when we remove all references to these structures? The answer is just algebra, representation theory and combinatorics. Loop quantum gravity II models are nothing but a general class of quantum theories constructed using only these ingredients. Consequently, one may think of generalized spin foams as a kind of generalized Feynman diagram, in which the momentum labels are replaced by representation of some algebra \mathcal{A} and the energy and momentum conserving delta functions at the nodes are replaced by invariants of \mathcal{A} .

5.4 Open questions in loop quantum gravity

Loop quantum gravity gives an apparently consistent microscopic descriptions of quantum spacetime, in both Hamiltonian and path integral forms. It is probably fair to say that no other approach to quantum gravity has amassed such a long list of highly non-trivial results concerning quantum spacetime at the background independent level. At the same time there remain important open problems.

The main open issues concern whether and how general relativity, coupled to quantum matter fields, is recovered in a suitable low energy limit.

For the case $\Lambda \neq 0$, there are good indications that an acceptable solution may be achieved, based on expansions around the Kodama state, as described in [108, 109, 111, 110, 38]. However, the question of whether or not the theory has a good low energy limit is open for general states. This includes the case $\Lambda = 0$ in which the Kodama state does not exist. This is a serious problem, because it is possible that a theory may be ultraviolet finite, but fail to have a phase in which there is anything like a low energy description described in terms of classical general relativity. This in fact occurs in some approaches to quantum

gravity such as, so far as is known, Euclidean dynamical triangulations in 4 dimensions¹⁴. Thus, if loop quantum gravity fails, this is likely to be how.

To study the problem of the low energy behavior, apart from the Kodama state, the following research programs are underway:

1) Renormalization group studies, based on a reformulation of the renormalization group to spin foam models. As a byproduct of this work it is shown that while the renormalization group is not a group, it does have a natural algebraic setting, as a Hopf algebra[125, 126].

2) The sums over labels in several spin foam models has been shown, to be convergent[127, 128]. This is surprising because the sum over labels is analogous to the momentum integrals in perturbative quantum field theory.

3) There is an understanding of coherent states of the quantum gravitational field, which is expected to play a key role in understanding the low energy limit within the Hamiltonian framework[122, 129].

It should also be emphasized that the question of whether a spin foam model has a good low energy limit should be asked, not just of quantum general relativity and supergravity in $3 + 1$ dimension (i.e. loop quantum gravity I), but for the whole infinite set of theories defined by loop quantum gravity II.

There are the following possibilities:

- A large class of loop quantum gravity theories have good low energy limits. In this case the existence of a low energy limit will be neither restrictive nor predictive.
- A restricted set, or possibly a unique, loop quantum gravity theory will turn out to have a good low energy limit. In this case the existence of a low energy limit will be predictive. For example, it is possible that only loop quantum gravity theories with non-vanishing values of Λ will have good low energy limits.

When a good low energy limit exists, it should be possible to discuss perturbation theory around it. Because studies of low energy excitations show that there are no perturbative states around a loop quantum gravity background with wavelength smaller than the Planck length, perturbation theory is expected to be finite. So far, however, no details have been worked out beyond the linearized states. So this remains an important open issue. One possible route towards its solution involves expanding around the Kodama state.

Another set of open issues is that of constructing Hamiltonians to get more detailed information about the dynamics of the hamiltonian theory. While it is important that there are many exact solutions to the full set of constraints, it is difficult to extract physics from most solutions because of problems constructing fully spacetime diffeomorphism invariant observables. One approach that could be developed more is to fix a time gauge, using either boundary conditions or matter fields to provide the definition of a clock, and construct the corresponding hamiltonians as operators on the space of spatially diffeomorphism invariant spin network states. While there have been a few papers about implementing asymptotically

¹⁴To be discussed in detail below.

flat boundary conditions more work needs to be done in this area as well. Another important step would be to extend the positive energy theorems from the classical to the quantum theory.

Another key open issue is the status of global lorentz invariance. We may note that there is no reason that quantum gravity must be Lorentz invariant, as this is only a global symmetry of a particular solution of the classical limit of the theory. Global symmetries are in no way symmetries of the fundamental theory of gravity, neither classically nor quantum mechanically. They are symmetries of particular solutions of the classical theory. Whether these symmetries are fully realized in the quantum states that have semiclassical approximations corresponding to these classical solutions is an open problem. The results mentioned suggest that the global lorentz symmetry is not fully realized in the ordinary way.

Indeed, as mentioned, several recent calculations indicate the presence of Planck scale corrections to the energy-momentum relations of the form of (1), effects that should be absent were the usual lorentz transformations exact symmetries[36, 37, 38]. One issue here is that different calculations make different assumptions about the ground state. In some the ground state is not Lorentz invariant, hence there is no surprise if the perturbations around them have non-lorentz invariant spectra. However, modified dispersion relations may also be seen by studying low energy excitations of a putative ground state that does not single out a preferred frame[38]. The question is then dynamical: can we determine the ground state precisely enough to discover whether the theory makes unambiguous predictions for the parameters in the energy-momentum relations (1)?

If these predictions survive further scrutiny, another important question is whether Scenario A) or case B) discussed in section 4 are realized. As we discussed there, not only does each possibility lead to effects that are observable in present or near future experiments, it appears possible that in Scenario A), some calculational results disagree with present observations.

If loop quantum gravity leads to Scenario A) it may then likely be ruled out as a quantum theory of gravity. There is, however, a simple reason why we would expect case B) to be realized. This is that, even though there is no global lorentz invariance in classical general relativity, the existence of effects due to a preferred frame are ruled out by the condition of invariance under the action of the hamiltonian constraint. This is because in any compact region the hamiltonian constraint can generate changes in slicing that in any finite region are indistinguishable from lorentz boosts. This is true even in the case of solutions, like homogeneous cosmological solutions, that have global preferred frames.

Now some of the key results of loop quantum gravity tell us that the hamiltonian constraint can be defined and solved exactly, and that no anomalies are introduced into the constraint algebra by the quantization. This makes it very probable that any quantum state that is both an exact solution to the hamiltonian constraint and has a semiclassical limit will in that limit describe physics which is to leading order invariant under the action of the classical hamiltonian constraint. This implies the absence of a preferred frame of reference in the classical limit of an exact solution to the hamiltonian constraint.

So this appears to rule out Scenario A), so long as the theory is defined in terms of

solutions to all the constraints. However there is no reason to expect global lorentz invariance must be realized as a linear rather than non-linear invariance. To the contrary there is a good physical reason to expect case B), which is that the Planck scale can be observer independent in the limit in which invariance under preferred frames is realized[31, 35].

Another set of open issues have to do with the physical inner product. The inner product on diffeomorphism and gauge invariant states is known exactly in terms of spin network states. In Thiemann's formalism[97, 122] the $SU(2)$ connection used is real[123] so the problem of realizing all the real observables as hermitian operators is solved. However, the inner product may have to be modified further to ensure that physical states, which are solutions to all the constraints, including the Hamiltonian constraint, are normalizable. A complete expression for the physical inner product is known in the spin network formalism[86]. However, it is unlikely to have a simple closed form. Thus a novel feature of spin foam models is that the physical inner product is incorporated in the path integral that defines the physical evolution amplitudes, and it has to be evaluated in whatever approximation scheme is being used to pull physical amplitudes from the spin foam path integral. Thus, while the solution to this problem is known in detail, it will be good to understand how it is implemented in detail in different expansions around non-perturbative states and histories.

There are also some unresolved issues concerning the role of the four dimensional diffeomorphism group in the Hamiltonian theory. This comes into the details of the regularization of the Hamiltonian constraint and the relationship between the hamiltonian and path integral quantizations. A set of related issues have to do with the relationships between the different forms of the quantum hamiltonian constraint arrived at by different regularization procedures and different operator orderings. We may note that the only necessary condition on a candidate form of the quantum hamiltonian constraint is that it have an infinite dimensional space of solutions, corresponding to a theory with an infinite number of degrees of freedom. This is satisfied by Thiemann's form of the constraint, and there is evidence that it is satisfied for the form of the constraint which is solved by the Kodama state. Further conditions have been suggested in the past, having to do with the algebra of the quantum constraints, however it seems impossible to implement them in a real quantum field theory where the constraints must be defined as limits of regulated operators.

Because we know that there are in fact infinite dimensional spaces of solutions to the constraints, none of these issues appears to be fundamental, but they need to be resolved nonetheless.

6 Definition and main results of string theory

6.1 The definition of a string theory

We cannot start off the discussion of string theory with a list of postulates, as we were able to do in the case of loop quantum gravity. The reason is that many string theorists would argue that, to the extent that string theory is the theory of nature, its postulates have not yet

been formulated. Moreover, the conceptual ideas and mathematical language necessary to express string theory in an axiomatic form are widely believed to remain so far undiscovered.

In this way string theory may be compared to previous research programs such as quantum mechanics and general relativity where several years of hard work preceded the formulation of the postulates of the theory. Thus, the *research program* called “string theory” can be taken to be a set of activities in search of the definition of a theory to be called “STRING THEORY.” What exists so far is only a collection of results concerning many different “string theories.” These are conjectured to be each an approximate descriptions of some sector of the so far undefined STRING THEORY. Thus, the discovery of the postulates of the theory is likely to occur close to the end of the development of the research program, it may indeed mark its culmination.

There is of course no *a priori* reason to believe such a research program will not pay off in the end. But this situation can complicate efforts to achieve a consensus or an objective evaluation of the status of the theory. For this reason I propose here to carefully separate results on the table from the exciting conjectures that have been made. Only by doing so can we get a good idea of what needs to be done to prove or disprove the main conjectures of the theory.

So my goal here will be to evaluate where string theory stands, *now*, with respect to its ability to answer the questions formulated in section 2. To discuss the results on the table, we cannot talk about STRING THEORY, for that does not exist as of this moment. We must instead talk about *string theories*, for these are what the results in hand concern.

Thus, by a string theory I will mean here what is sometimes called a perturbative string theory. A more accurate name, which I will use here, is a background dependent string theory. These are theories which are defined in terms of the embedding of two and higher dimensional quantum extended objects in a background classical spacetimes. So far as I have been able to determine, all of the firm and widely accepted results of string theory concern such background dependent theories.

To define a background dependent string theory, one must first specify a classical background, consisting of a given manifold \mathcal{M} , of some dimension d and a metric, g_{ab} . The background fields are often supplemented by certain other fields, which include a scalar field Φ , called the dilaton, and generalizations of electric and magnetic fields, which we will denote generally as A . We then denote a choice of background $\mathcal{B} = \{\mathcal{M}, g_{ab}, \Phi, A\}$.

There are classical theories of the motion of strings, as well as higher dimensional membranes in such backgrounds. Examples include theories of the stretched strings and membranes used in musical instruments. But what makes string theory challenging is that not all such theories can be cast into the domain of quantum theory. In many cases inconsistencies appear when one attempts to describe a string or membrane stretched in a classical background in the language of quantum mechanics.

But not always. What is remarkable is that there are some string theories which appear to be consistent quantum mechanically. They are what the subject of string theory is all about.

Thus, the important definition to make is that of a *consistent string theory*. This is

defined as follows:

- A consistent string theory is a quantum theory of the propagation and interactions of one dimensional extended objects, closed or open, moving in a classical background, \mathcal{B} , which is completely consistent quantum mechanically. In particular it is unitary (which means quantum mechanics preserves the fact that probabilities always add up to one) and the energy is never negative[4, 5].
- A background \mathcal{B} is called consistent if one may define a consistent perturbative string theory moving on it. Many backgrounds are not consistent. However there are a very long¹⁵ list of consistent backgrounds, and some backgrounds allow more than one perturbative string theory to be defined on it.
- Consistent string theories are generally characterized by two parameters, which are a length l_{string} , called the string scale, and a dimensionless coupling constant g_{string} , called the string coupling constant. There may also be additional parameters associated with the different backgrounds. These measure aspects of their geometry or the values of the other background fields. In many cases these may be varied continuously without affecting the consistency of the string theory.
- A string theory is called *perturbative* if it describes interactions of strings in terms of a power series in the dimensionless coupling constant g_{string} , such that when $g_{string} = 0$ there are no interactions.

Now we turn to the results.

6.2 Basic results of perturbative string theory

1. Perturbative string theories are known, which are consistent through second non-trivial order in string perturbation theory¹⁶[4, 5]. These include five supersymmetric string theories which are defined in 10 dimensional Minkowski spacetime.

The one loop consistency is well understood[4, 5], while the two loop consistency has only been proved recently[130]. Beyond two loops, there are partial results[131] which support the conjecture that the theory is consistent to all orders. There are intuitive arguments that suggest that ultraviolet divergences of the kind that plague conventional quantum field theory cannot occur in string theory. The main reason is that the interactions of strings involve the breaking and joining of strings and these do not take place at points. However, a string theory can fail to be consistent for other reasons. There may be infrared divergences, or ambiguities in the definition of the amplitudes, there can be anomalies in the action of the lorentz boosts, or the theory may fail to be

¹⁵counting distinct classical backgrounds as distinct.

¹⁶String perturbation theory is defined as an expansion in the genus of the topology of the two dimensional world surface of the string. Leading order is a sphere, first leading order is a torus, etc.

unitary. The problem of the consistency of perturbative string theory appears to be very challenging, and the proof of consistency at two loops required a very impressive technical *tour de force*[130].

As it does not appear to be widely appreciated that the consistency of string perturbation theory is still open, I quote here from a recent paper by experts in the field, which announced the proof of consistency at the two loop level.

Despite great advances in superstring theory, multiloop amplitudes are still unavailable, almost twenty years after the derivation of the one-loop amplitudes by Green and Schwarz for Type II strings and by Gross et al. for heterotic strings. The main obstacle is the presence of supermoduli for world-sheets of non-trivial topology. Considerable efforts had been made by many authors in order to overcome this obstacle, and a chaotic situation ensued, with many competing prescriptions proposed in the literature. These prescriptions drew from a variety of fundamental principles such as BRST invariance and the picture-changing formalism, descent equations and Čech cohomology, modular invariance, the light-cone gauge, the global geometry of the Teichmüller curve, the unitary gauge, the operator formalism, group theoretic methods, factorization, and algebraic supergeometry. However, the basic problem was that gauge-fixing required a local gauge slice, and the prescriptions ended up depending on the choice of such slices, violating gauge invariance. At the most pessimistic end, this raised the undesirable possibility that superstring amplitudes could be ambiguous, and that it may be necessary to consider other options, such as the Fischler-Susskind mechanism[130].

This situation is a bit disappointing, given that the main claim for string theory as a quantum theory of gravity is that it alone gives a consistent perturbation theory containing gravitons. After all, supergravity theories, which are ordinary field theories which extend general relativity to incorporate supersymmetry, are also consistent in perturbation theory at least to the two loop level and $N = 8$ supergravity in four dimensions is expected to be consistent at least to five loops[132]. The difference is that there are reasons to expect that supergravity theories become inconsistent at some point beyond two loops, while no reason is known that the technical difficulties that have blocked a proof of the consistency of perturbative string theory cannot someday be overcome.

It is further known that bosonic string perturbation theory is not Borel resumable and this is conjectured to extend to superstring theory[133]. This means that the theory cannot be defined completely by perturbation theory because there may be excitations of the full theory that are not captured in the perturbation theory.

From now on, when I label a string theory “consistent” I really mean that it is known to

be consistent to one or two loop order, and that no reason is known why the conjecture of all orders consistency may not apply to it.

2. There are consistent string theories with spatially closed boundary conditions (so the string is a closed loop) and with open boundary conditions. The latter, called open string theories have spectra that include the quanta of Yang Mills fields[4, 5]. The Yang Mills coupling g_{YM} is related to the string coupling by $g_{YM}^2 = g_{string}$ ¹⁷.
3. Consistent closed string theories have spectra that include massless gravitons propagating on the background[4, 5]. They couple with a Newton's constant given by¹⁸ $G_{Newton} = g_{string} l_{string}^2$.
4. All string theories which are known to be consistent to second order are supersymmetric. There are some non-supersymmetric string theories which appear to be consistent at least to first non-trivial order. While not supersymmetric, these have spectra in which fermions and bosons are grouped in pairs of equal mass[134]. As this is not a feature of our world, if string theory is true it must be that supersymmetry (or at least fermi-boson matching) is spontaneously broken.
5. Many known consistent string theories have backgrounds that are 10 dimensional manifolds, or can be understood as arising from 10 dimensional manifolds by compactifications and identifications[4, 5]. A simple class of example of such consistent compactified backgrounds are those cases in which the compactified d dimensional manifold is a d -torus.
6. A necessary¹⁹ condition for a perturbative string theories to be consistent is that the two dimensional world sheet quantum field theory that defines the theory be conformally invariant[4, 5, 135]. This means that the conformal anomaly on the two dimensional worldsheet vanishes. To leading order in l_{string} this condition is equivalent to the Einstein equations of the background manifold²⁰.
7. There are a very large number, perhaps infinite, of backgrounds of the form of four dimensional flat spacetime producted with a six dimensional compact manifold. A large class exists where the six dimensional compact manifold is a Calabi-Yau manifold. There are estimated to be on the order of at least 10^5 distinct such manifolds.

¹⁷In four dimensions where the Yang-Mills coupling is dimensionless.

¹⁸Again, in $d = 4$.

¹⁹There is a variant of string theory, called non-critical string theory [136], in which the conformal anomaly does not vanish perturbatively, but it is claimed, under certain conditions, it may vanish as a consequence of a non-perturbative regularization scheme. The literature on this subject describes some interesting ideas, however I do not understand the status of the claims made well enough to include them in this review.

²⁰Corrections to the background field equations have been worked out.

8. Most of the known backgrounds of the form of $Mink^{10-d} \times Compact^d$ have parameters that measure the geometry of the compactified manifold, in many cases these parameters can vary over the $10 - d$ dimensional manifold and hence become scalar fields, called moduli fields, on the $10 - d$ dimensional manifold. In many cases the energy does not depend on the values of the moduli parameters, so the fields that represent them on $Mink^{10-d}$ are massless.

No consistent string theories are known that have no massless scalar fields.

9. These include the case $10 - d = 4$, which is so far at least supported by all observations, but consistent backgrounds exist for any d up to 9.
10. There are transitions in string theory in which the topology of the d dimensional compact manifold changes[137].
11. There are consistent string backgrounds for 4 large, uncompactified dimensions that correspond to a large range of possible values for the number of generations, for the number of Higgs fields and for the gauge group. Thus, string theory makes no prediction for these characteristics of the standard model[138]. (See figure 1.)
12. There are so far not known any consistent, stable, string backgrounds of the form of DeSitter spacetimes times a compact manifold[139, 140]²¹.
13. More generally all known consistent, stable string theories have time-like killing fields. No consistent stable string backgrounds are known which are time dependent²².
14. There are an infinite number of consistent backgrounds that include structures known as D -branes[5]. These are submanifolds of various dimensions embedded in the background on which open strings may end. These branes may be charged, with respect to some of the generalized electric and magnetic fields. The D -branes have dynamics induced by their coupling to the strings and charges. This includes Yang-Mills fields which propagate on the branes[141, 142].
15. By a careful choice of arrangements of several branes, intersecting at carefully chosen angles, one can construct a string theory background whose low energy limit has some features of the supersymmetric standard model, including chiral fermions and parity

²¹The reason is related to the fact that there is no unitary representation of the supersymmetric extension of the symmetry group of deSitter spacetime. There have been intriguing suggestions from string theorists about quantum gravity with a positive cosmological constant, a few of them involve string theory explicitly. However Ed Witten recently wrote, "In fact, classical or not, I don't know any clear-cut way to get de Sitter space from string theory or M-theory. This last statement is not very surprising given the classical no go theorem. For, in view of the usual problems in stabilizing moduli, it is hard to get de Sitter space in a reliable fashion at the quantum level given that it does not arise classically[139]."

²²This limitation is tied to supersymmetry, because the closure of even the $N = 1$ supersymmetry algebra contains the hamiltonian, and that is only well defined on backgrounds that possess a timelike killing field.

violating gauge couplings[143]. However, many other consistent backgrounds exist which do not have these features[138].

6.3 Results and conjectures concerning black holes

String theory also has led to results which are relevant for the understanding of black holes. To express them one has to know that in the state space of a supersymmetric theory there is a subspace in which a fraction of the supersymmetry transformations are broken, leaving still unbroken a number of supersymmetries at least twice the dimension of the spinors in that dimension. These are known as *BPS* states and they have special properties[144]. In particular, aspects of the spectra of the Hamiltonian are strongly constrained by the supersymmetry algebra. One way to characterize *BPS* states is that they are states in which certain of the generalized electric and magnetic charges are, in appropriate units, equal to their masses.

Classical supergravity has *BPS* states (i.e. classical solutions), among which are black holes whose charges are equal to their masses[144]. These are also called extremal because there is a theorem that the charges cannot exceed their masses. This indeed suggests a relationship between supersymmetry and the properties of black holes. The extremal black holes have zero Hawking temperature but nonzero Bekenstein entropy[144].

The results in string theory do not concern, precisely, black holes, as they are found in a limit in which the gravitational constant is turned off. But they concern systems with the same quantum numbers as certain black holes, which, it may be argued, may become black holes if the gravitational constant is turned up to a sufficiently strong value. Still they are very impressive,

1. For certain compactifications, with $d = 3, 4$ or 5 flat directions, and in the limit of vanishing g_{string} , and hence G_{Newton} , there are *BPS* states of string theory including *D*-branes, which have the same mass, charges and angular momenta of an extremal black hole in d dimensions. The number of such states is in all cases exactly equal to the exponential of the Bekenstein entropy of the corresponding black hole[145, 146, 147].
2. If one perturbs away from the *BPS* condition for the string theory states, to a near extremal condition, and constructs a thermal ensemble, the spectrum of the Hawking radiation from the corresponding near extremal black hole is reproduced exactly, including the grey body factors[147].

These results are very impressive; the agreement between the formulas obtained for entropy and spectra between the *D*-brane systems and black holes are staggeringly precise. It is hard to believe that this level of agreement is not significant. At the same time, there are two big issues. First the *D*-brane systems are not black holes. Second it has not been found possible to extend the results away from the neighborhood of extremal, *BPS* states, so as to apply to ordinary black holes.

We are then left with a conjecture:

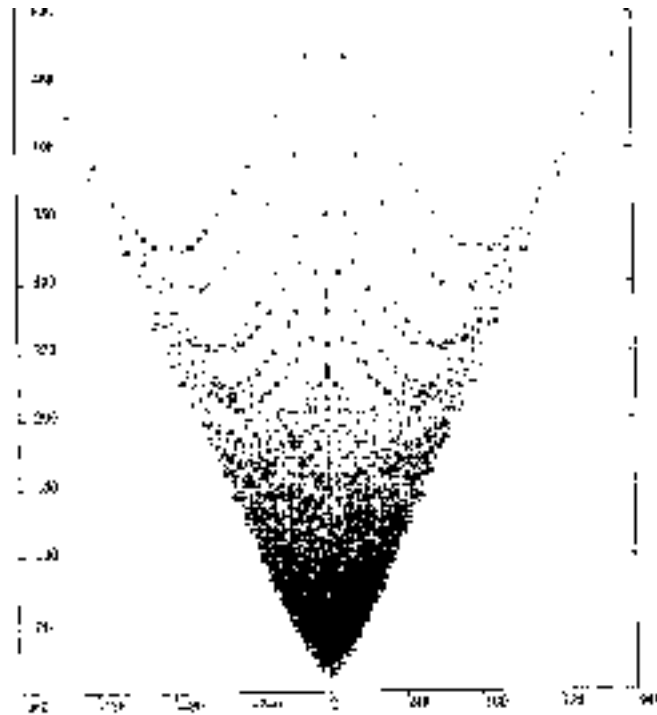


Figure 1: A sample of the phenomenology predicted by different consistent string theories. The vertical axis is the number of Higgs fields, up to 480, the horizontal axis is related to the number of left handed fermion fields minus the number of right handed fermion fields. According to string theory we could equally well live in any of these universes. From [138].

- **Black hole conjecture.** If one turns the gravitational constant up in the presence of a thermal ensemble of states which as described above, reproduce the entropy and temperature of an extreme or near extremal black hole, one can construct a string theoretic description of quantum black hole spacetimes. This will extend also to far from extremal black holes.

We may note that some arguments, non-rigorous but physically motivated nevertheless, support the conjecture that the states in string theory which correspond to a Schwarzschild black hole have an entropy proportional to the square of the mass[176, 167]. However the constant of proportionality is so far not predicted.

6.4 Results and conjectures concerning dualities

In section 3 I emphasized the importance of the notion of duality in both string theory and loop quantum gravity. There are indeed a number of very interesting results concerned with how duality is realized in string theory. These motivate a number of conjectures which, if true, are quite important for the physical interpretation of string theory. As some of the conjectures remain unproven, however, it is important to distinguish results from conjectures.

6.4.1 T duality

In compactifications on tori, or more generally where the compact manifold has non-trivial π^1 , the string theory spectrum has states distinguished by a winding number around a circle as well as the usual vibrational modes. In all these cases there is a symmetry in which one exchanges winding and vibrational modes and, in units of the string scale, l_{string} , takes the radius of the circle, R to l_{string}^2/R .

T -duality appears to be a general property of string theories[4, 5]. It depends neither on supersymmetry nor on criticality and so appears to be true for all string theories. However, the next two cases require some care as there are some unproven conjectures.

6.4.2 S duality

S duality is inspired by the old observation, which goes back at least to Dirac, that electromagnetism is almost invariant under an exchange of electric and magnetic fields. The idea is that if we had a theory with magnetic monopoles, with magnetic charge g as well as ordinary particles with ordinary electric charge e , then the theory of Maxwell, modified to include the magnetic monopoles, appears symmetric under an exchange $e \leftrightarrow 2\pi/g$. Thus, if

$$g = 2\pi/e \tag{3}$$

the theory might be symmetric under the symmetry operation in which $e \rightarrow 1/e$ and electric charges and magnetic monopoles are exchanged.

In certain supersymmetric Yang-Mills theories, this appears to be the case, at least to some approximation[148]. This is because the theory has solitons which are magnetic

monopoles which satisfy eq. (3). There is one theory, the $N = 4$ supersymmetric Yang-Mills theory in 4 dimensions in which there is good evidence for this at least in the BPS sector of the theory²³.

In string theory there are several related results.

- For many string backgrounds \mathcal{B} there is an S -dual background \mathcal{B}' such that the free string spectra and BPS subspace on \mathcal{B} can be mapped onto the free spectra and BPS subspace on \mathcal{B}' with g_{string} taken to $1/g_{string}$ [149].
- In some cases \mathcal{B} and \mathcal{B}' are the same, and the duality maps the BPS sector of a single theory to itself. In this case we may speak of the theory being self-dual, at least on the BPS sector.

This is quite an impressive fact, as it tells us that there are indeed theories, in which a generalization of electro-magnetic duality holds exactly, at least in a sector of the state space. It is then very interesting to ask whether the duality transformations hold exactly in string theory only on the BPS sectors of the theory, or extend to the full theories in question.

The answer depends on whether the existence of the duality is a consequence of the BPS conditions, or is an expression of a deeper property of the dynamics of string theory. It is true that the supersymmetry algebra strongly constrains the spectra and degeneracies of the BPS sector, because the Hamiltonian is part of an algebra that generates the spectrum. If this is all there is to it, it is an impressive result, but it would not be expected that the duality would apply to the whole theory.

However many string theorists believe that S duality is a general property of string theories, This conjecture may be stated as

- **S duality conjecture.** Whenever an S duality exists in the BPS sector of a string theory it extends to an isomorphism on the full Hilbert spaces of the theories in question.

There are a few results in string theory concerning the spectra of non- BPS states, and they do show that a duality transformation continues to exist, at least approximately, to leading order in departures from the BPS condition[150]. Were this not the case the S duality conjecture would be falsified. At the same time, to my knowledge, there is no stronger result and no proof supporting the S duality conjecture in string theory.

6.4.3 String/gauge theory dualities

These are a new kind of duality which connects, not different string theories, but string theories and gauge theories. They are very reminiscent of the original ideas of duality.

²³Very recently there are results which strongly support the conjecture that supersymmetric Yang-Mills theories are S -dual[187] to all orders in perturbation theory.

However this idea has been realized in the last few years in a novel way, in which a string theory in d non-compact spacetime dimensions is related to a gauge theory in $d-1$ spacetime dimensions.

The reason for the difference in dimensions can be explained by a remarkable argument, due originally to Polyakov[151]. He observed that a string theory may be expressed as a two dimensional quantum field theory on the two dimensional worldsheet of the string. Among the fields that live on the worldsheet are the imbedding coordinates, $X^a(\sigma)$, where X^a are d coordinates in the d -dimensional background spacetime and σ are the two dimensional coordinates on the worldsheet. However, Polyakov noted that to make the worldsheet theory one needs also the metric on the worldsheet $h_{\alpha\beta}$, so as to form the action,

$$I = \int d^2\sigma \sqrt{h} h^{\alpha\beta} (\partial_\alpha X^a) (\partial_\beta X^b) g_{ab}(X(\sigma)) \quad (4)$$

Here g_{ab} is the metric of the background spacetime.

The two dimensional coordinates of the worldsheet can be fixed so that

$$h_{\alpha\beta} = \eta_{\alpha\beta} e^\phi. \quad (5)$$

where η is the metric of flat, two dimensional spacetime. This leaves unfixed the third component of the metric, represented by ϕ , which we see here is the conformal factor.

Polyakov noticed that the quantization of the X^a fields on the worldsheet will in general give rise to a conformal anomaly. This will occur in spite of the fact that the classical action is conformally invariant. This gives rise to a dynamics for the ϕ field of the form

$$I' = \hbar \int d^2\sigma \sqrt{h} h^{\alpha\beta} (\partial_\alpha \phi) (\partial_\beta \phi) \dots \quad (6)$$

Thus, if this conformal anomaly is not cancelled it is as if the string moves in an $d+1$ dimensional spacetime background, whose coordinates are X^a, ϕ .

Now for what are called critical string theories, the anomaly is cancelled by factors coming from the ghosts, which must be there in turn because of the gauge fixing down to the gauge (5). The resulting string theories have massless degrees of freedom. However, for some gauge theories, like *QCD* we do not expect there to be massless gluons in their spectra, due to confinement or, in some cases, to a Higgs effect. For such theories there cannot correspond a critical string theory. However, there are still general arguments, based on the idea of duality, that suggest that some kind of string theory should be related to any gauge theory. The resolution of this puzzle is that for such gauge theories there should correspond a non-critical string theory. But then Polyakov's observation suggests that the dual string theory should then appear to live in a spacetime of one additional dimension.

Moreover it is not hard to see that if the background spacetime is a Minkowski spacetime, the spacetime of one higher dimension that is created is an anti-deSitter (AdS) spacetime. The original Minkowski spacetime can be thought of as part of the boundary of that Anti-deSitter spacetime[151].

Thus, the suggestion is that to any non-Abelian gauge theory that does not have massless gluons in its spectra in d dimensional Minkowski spacetime there should correspond a string theory in a $d + 1$ dimensional anti-deSitter spacetime.

There is a further argument that suggests a relationship between quantum field theories on $Minkowski^d$ and on AdS^{d+1} . This is that the symmetry group of AdS^{d+1} spacetime is the same as the conformal group on $Minkowski^d$. This suggests the relationship between a gauge and a string theory on these two spacetimes should be especially tight in any case in which the gauge theory on $Mink^d$ is conformally invariant.

Now *classical* gauge theories in $d = 4$ are conformally invariant so long as they don't couple to massive fields. But in general the conformal invariance is broken by quantum corrections. However there are a few cases in which supersymmetric gauge theories are known, in perturbation theory, to have vanishing β functions, which means that they are, in perturbation theory, exactly conformally invariant. One of these is the most supersymmetric non-gravitational theory that exists in four spacetime dimensions, the so called $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. It is then natural to guess that in such cases there could be found interesting results connecting them to string theories.

Arguments and conjectures concerning a possible connection between string theory and supersymmetric Yang-Mills theory were put forward in 1997, first by Maldacena[152], Witten[153], and Gubser, Klebanov and Polyakov[154]. Since then a large number of very interesting results have been found in this direction. While it is without doubt that these results are highly significant, this is also an area in which it is necessary to distinguish results from conjectures. Let us start with the results²⁴,

- **Conformal induction[153, 156].** Consider a quantum theory, T_1 defined by a path integral, on a background \mathcal{B} whose spacetime is of the form of \mathcal{M}^d where \mathcal{M}^d is either an anti-deSitter spacetime, or a more general spacetime that is asymptotically anti-deSitter. Then it can be shown that this spacetime has a boundary whose timelike component is isomorphic to M^{d-1} where M^{d-1} is flat spacetime. One can then argue generally that one can define a quantum field theory on the boundary by evaluating expectation values of local operators in the theory T_1 in which all the operators are taken to the boundary. One can also argue that this new theory must be conformally invariant (or more specifically have exact scale invariance, with perhaps spontaneously broken conformal invariance.)

Furthermore the same holds for quantum theories defined on spacetimes of the form given, produced by a compact manifold.

Thus in these cases a conformal field theory on the boundary is defined from a subset of the observables of the bulk theory, those in which N point functions for fields are evaluated all on the boundary. We may say that the boundary theory is *conformally induced* from the bulk theory. In such cases, some of the observables of the bulk

²⁴The argument of this section was developed in collaboration with Matthias Arnsdorf. A more detailed version of this argument is presented in [156].

theory are computable in terms of N -point functions of the boundary theory. But in the general case there is no reason to believe that the two theories are isomorphic, for there will generally be observables of the bulk theory that are not computable in terms only of N -point functions on the conformal boundary. One reason is that there are components of the boundary of an AdS spacetime besides the timelike component which is isomorphic to M^{d-1} . This includes future and past timelike infinity.

- There are many results that suggest that the conformal induction of linearized supergravity on $AdS^5 \times S^5$ is a certain limit of the $N = 4$ supersymmetric Yang-Mills theory on four dimensional flat spacetime[153, 155].
- There are additional results that strongly indicate that the conformal induction of supergravity on various asymptotically AdS^5 spacetimes is related in this way to $N = 4$ super-Yang-Mills theory by reduction or symmetry breaking[155]. Among them are results that are consistent with the conjecture that the linearization of supergravity on the AdS-Schwarzschild solution induces a thermal state in the Yang-Mills theory[157].
- Still other results of this kind hold for other d .
- It is not, however, completely established whether perturbative string theory is well defined on an $AdS^5 \times S^5$ background, beyond the supergravity approximation. While the classical action for a free string on the $AdS^5 \times S^5$ background has been constructed[159], there is no gauge in which it is a free theory. Thus the free string theory cannot be solved exactly on an $AdS^5 \times S^5$ background, as it can be in the case of flat ten dimensional spacetime. Instead, to define the free string theory one has to treat it as an interacting two dimensional quantum field theory, defined on the string world sheet[158].

The resulting theory has been studied and, in some particular examples, shown to be a conformal field theory at least the one loop level[160]. There are also arguments that the theory remains a conformal field theory to all orders[161, 160]. These results are very reassuring, but we are still apparently lacking a general proof of the consistency of the interacting string theory, as is possible on flat space through at least the two loop order.

- A certain limit of $AdS^5 \times S^5$ is known which is a plane wave[162]. This is gotten by expanding the metric around a null geodesic that circles the sphere, while remaining at the center of the AdS^5 . In this case the free quantum string theory can be constructed and solved explicitly. The analogous limit can be constructed in the $N = 4$ super Yang-Mills theory. The resulting spectra matches that of the string theory on the plane wave.
- There is a general result of axiomatic quantum field[163] theory that, given an exactly conformally invariant quantum field theory, without anomaly, on $Minkowski^d$, constructs an axiomatic quantum field on AdS^{d+1} . This result is rigorous, but it requires

that the theory on *Minkowski*^{*d*} have no anomaly in any of the generators of the conformal group. However, the supersymmetric Yang-Mills theory do not satisfy this, at least in perturbation theory[164]. While they have $\beta = 0$, and hence no anomaly in the action of dilations, they have anomalies in the action of the large conformal transformations. Hence it appears that the gauge theory cases studied are not examples of this particular version of the correspondence.

As in the case of *S* duality there are conjectures which many string theorists believe which, if true, greatly extend these results.

- **Maldacena conjecture.** There is an isomorphism between $N = 4$ supersymmetric Yang-Mills theory in $d = 4$ and “string theory on $AdS^5 \times S^5$.” This is sometimes called the Maldacena conjecture[152].

There are actually a number of different conjectures that are often conflated in discussions. The Maldacena conjecture, as I have stated it, is the strongest of them²⁵.

Even stated as such, one might mean two different things by the Maldacena conjecture.

- **Maldacena I.** String theory “on an $AdS^5 \times S^5$ background” can be given a precise, consistent non-perturbative definition, as can $N = 4$ supersymmetric Yang-Mills theory in 4 dimensions. After they are both defined, each independently of the other, it can be shown that they are isomorphic.
- **Maldacena II.** String theory “on an $AdS^5 \times S^5$ background” can be given a precise, consistent non-perturbative definition *by assuming a certain correspondence between it and $N = 4$ supersymmetric Yang-Mills theory.*

The first thing that must be said in evaluating the evidence for either of these conjectures, is that they are each logically much stronger than the conjecture of conformal induction, as I stated it above. In fact conformal induction is a very general property, and it may be supported by general arguments that assume only the existence of a bulk and boundary theory on an *AdS* or asymptotically *AdS* spacetime. These arguments do not assume any special properties of supersymmetric theories or gravitational theories and, indeed, there are known examples where such a correspondence holds for non-gravitational and non-supersymmetric theories.

There is unfortunately a lot of confusion about this in the community, due perhaps to the fact that the papers of Witten and Klebanov et al that followed Maldacena conjecture were mainly concerned with presenting arguments for the weaker conjecture of conformal induction. As a result, many members of the string theory community appear not to have noticed, or not to be concerned with the fact, that the conjectures are very different, and have rather different implications.

²⁵For more details on the different possible versions of the Maldacena conjecture, see [156].

Many results have been obtained supporting some version of a correspondence between quantum theories in AdS spacetimes and gauge or conformal field theories on their boundaries. Given the existence of different conjectures, care is required in evaluating which conjectures are supported by which results.

When carrying out this analysis, it is important to remember a basic point of logic. In a case in which two conjectures are each consistent with a given set of results, and one conjecture is logically stronger than the other, it follows that the evidence may be taken to support only the weaker conjecture. Only results which are not consequences of the weaker conjecture may be taken as evidence for the stronger conjecture²⁶.

In fact, almost all of the results found concerning an AdS/CFT correspondence are explained by the weaker conjecture of conformal induction. For example, all of the results concerning matching N point functions between classical supergravity and the quantum Yang-Mills theory are of this kind, as are the results concerning matching (up to overall constants) of entropies for thermal states[156].

In discussing this situation one may ask the following question. Suppose that it turns out that there is no consistent *interacting* string theory on an $AdS^5 \times S^5$ background²⁷. Would this contradict any of the results so far found which are used as evidence for an *AdS/CFT* correspondence?

If the answer is no then that evidence is completely consistent with the possibility that the strongest general result which holds between the gauge theory and the gravity theory is a form of conformal induction. Further, as no interacting string theory may in this case exist, it holds only between either the free string theory, or supergravity, expanded on the $AdS^5 \times S^5$ background and a certain limit of the $N = 4$ supersymmetric Yang-Mills theory.

Of interest in this regard is the recent work connecting the plane wave limit of $AdS^5 \times S^5$ with a similar limit in the supersymmetric Yang-Mills theory[162]. This work is extremely interesting and introduces novel techniques which illustrate how a string theory may arise from a gauge theory. It is then of great interest to establish which version of an *AdS/CFT* connection is supported by these results. The key question is whether these results are implied by a combination of conformal induction and the *BPS* conditions. If they are then, as interesting as they are, these do not support the Maldacena conjecture over conformal induction. One might think that conformal induction is not involved, as the limit taken involves expanding the spacetime around a trajectory far from the boundary. However, the *BPS* conditions do play a role in the derivation, so it is possible that the agreement found is simply a consequence of the fact that the same supersymmetry algebra acts in the plane wave spacetime as on the related limit of the boundary gauge theory, and the correspondence

²⁶Some readers have questioned this principle of logic, so let me give a simpler application of it. So far the friends I had in high school are all, to my knowledge still alive. There are two possible conjectures that would account for this observed fact. The first is that we are still much younger than our common statistical life expectancy. The second is that we are all immortal. The second is stronger than the first, because it logically implies it. No amount of wishful thinking will convert what is decent evidence for the weaker conjecture into evidence for the stronger conjecture.

²⁷Not that there is reason to think this is the case, but just to make the following argument.

found is a consequence only of the fact that the extended supersymmetry algebra constrains some feature of the spectra of the theories. Results which settle this issue would clearly be of interest.

In any case, apart perhaps from the Penrose limit, there appear to be so far no results relevant for interacting strings on the $AdS^5 \times S^5$ background. This means that it is not yet possible to claim evidence any conjecture stronger than conformal induction, except perhaps in the Penrose limit.

A final point that must be made concerns the status of $N = 4$ supersymmetric Yang-Mills theory. That theory is well defined perturbatively, and it is known to have a vanishing β function. There is also evidence that it is exactly S -dual to all orders in perturbation theory.

This suggests two beautiful conjectures. First that this theory is well defined also non-perturbatively. Second, that it is an exactly scale invariant quantum field theory. However, as of this date, the theory has not been given a non-perturbative definition. The usual route to a non-perturbative definition of a gauge theory is through a non-perturbative regularization such as the lattice theory. Given the fact that the β function vanishes, it is to be expected that *were such a non-perturbative regularization constructed*, it would be easier to prove the existence of the theory in the limit that the regulator is removed than in the ordinary, non-supersymmetric case. However, all known non-perturbative regularizations of gauge theories, including lattice methods, break supersymmetry. Hence, so far, we do not have at our disposal any non-perturbative definition of $N = 4$ supersymmetric Yang-Mills theory²⁸

Since the theory itself has not yet been defined non-perturbatively, it follows that it cannot be used to provide a non-perturbative definition of another theory. Thus, while it may be that in the future this situation is improved, for the time being it is not true that the Maldacena conjecture has been proved. Nor can it be said that the problem of giving a background independent or non-perturbative definition of string theory has been solved by assuming the conjecture is true and therefore defining string theory non-perturbatively in terms of the $N = 4$ supersymmetric Yang-Mills theory.

6.5 Open issues of string theory

It is clear from the summary just given that there is very good reason to take string theory seriously. The theory appears to give a good perturbative description of quantum gravity through at least the two loop level and, even if this is true also of supergravity theories, this is still a very impressive fact. Many of the results are very impressive, including the ones which show that there exist analogous systems in string theory with the same entropies and temperatures of extremal and non-extremal black holes.

At the same time, there are a large number of open issues.

²⁸Recently it has been suggested that the correspondence just discussed may suffer from a fermion doubling problem[189], and that this implies either a failure of the correspondence or a breaking of supersymmetry in the $N = 4$ supersymmetric Yang-Mills theory beyond perturbation theory. My understanding is that this issue is presently unresolved.

- There are a very large number of string theory backgrounds, labeled by both discrete topological classes and continuous parameters.
- So far, no string theory background is known which is consistent with all features of the observed universe. They all have one or more of the following features, which each disagree with observation: no positive cosmological constant, unbroken supersymmetry, massless scalar fields.
- So far no string theory background is known which is time dependent, as is our universe. Further no stable string theory background is known which is consistent with recent observations that strongly suggest that there is a positive cosmological constant[139].
- Further, we observe no massless scalar fields. Thus in any string background corresponding to nature there can be no such fields. This means that the compactified geometry must be a consistent background only for discrete values of its parameters. At the same time, those parameters must have very small ratios in them, to explain the hierarchy problem.
- Even if a string theory background is found which is also consistent with everything that is observed, does this tell us anything, given that there is an infinite space of possible string backgrounds to search? The theory would be predictive only if there were a unique string background consistent with what is observed. Is there any reason to believe this is the case, rather than there being a large or infinite number of such backgrounds?

So, we should ask, even if there is a unique string theory background consistent with what is observed, how would nature pick it out? One might hope that there were a principle of stability or lowest energy that would pick out a unique string theory background. However this is unfortunately unlikely. We have good reason to believe²⁹ that many of the supersymmetric vacua are stable. So it appears very unlikely that the observed background is the only stable one.

Thus, even if string theory is true, there is so far no reason to believe that nature has a unique choice as to low energy phenomenology. Whatever the hopes, present evidence from string theory is more compatible with the idea that the observed background is picked out from many possible consistent ones by some dynamical process, occurring in the early universe, or even before the big bang³⁰.

This circumstance suggests that perhaps some attention be given to what might be called the search question in string theory. Given what we know, it is likely that if string theory is true, the real world is described by one out of a very large number of

²⁹From results that support the conjecture of *S*-duality.

³⁰One such theory is cosmological natural selection[165, 64], which was invented to address this issue in string theory. It remains falsifiable, but so far it has not been falsified and is consistent with all observations to date.

local minima of some potential or energy functional. This may be either a function on the space of string backgrounds or the expectation value of a potential or Hamiltonian in some fundamental Hilbert space of string theory. In either case, we have to find the global minimum of a function on a high dimensional space, which parameterizes possible string backgrounds. Further, we expect that the function has many local extrema, corresponding to the perturbatively consistent string theories. How do we find its global extremum?

Further, if the global minimum is to agree with observed physics, there can be no massless moduli fields. This means that the true minimum must be an isolated point in the space of consistent string theories. However, all known consistent string theories have massless scalar fields, this means that all the local minima that correspond to them live in continuous submanifolds of solutions.

How then are we to find the one true, isolated minima of a very complicated potential, which we know has lots of other minima, many of which have much more measure than the solution we seek?

It is fair to ask whether examining them one by one, as they are discovered by people putting together ever more complicated combinations of branes, orbifolds and complex manifolds is likely to hit on the true one. After all, the number of consistent backgrounds vastly outnumbers the number of people working in the field. Should we be concerned that picking out the true minimum of a complex potential with a large number of local minimum is known from results in the theory of computation to be a very hard problem?

A striking result of complexity theory is somewhat worrying in this regard. Called the “no free lunch theorem” this states that no specific search algorithm is likely to do better than random search in finding the global minimum of a randomly chosen complicated potential[166]. To do better than random search, a search procedure must be based on an algorithm which is crafted taking into account some properties of a given potential.

This suggests that if we are ever to find the string vacua that describes our world we need to craft a search algorithm based on some non-trivial property of string theory, rather than just studying more and more complicated string vacua as the tools are developed to define them.

- It must also be emphasized that string theory does not give a genuine quantum theory of gravity, in the sense that each consistent string theory is defined with respect to a fixed, classical, non-dynamical background. So it is not background independent and it fails to address many of the questions that a quantum theory of gravity must answer³¹.

³¹Some string theorists have argued that the Maldacena conjecture offers a non-perturbative definition of string theory that is background independent in the sense that all observables of quantum gravity are mapped into observables of the supersymmetric Yang-Mills theory. The only restriction is that the quantum

Related to this is the fact that the black hole results in string theory do not concern actual black holes. They concern instead ensembles of states in free string theory in flat spacetime, with the gravitational coupling turned off. So far, no way has so far been found to extend the results to black holes that are not extremal or near extremal³².

To address all these issues, a number of conjectures have been made. Some of these date back to the early days of string theory, 1984-5, and have been outstanding ever since. Several others have been added more recently.

6.6 Open conjectures of string theory

We have already mentioned four conjectures in string theory, so far unproven. These were

- **Perturbative superstring theory is finite, unique and consistent to all orders in the genus expansion.**
- **The black hole conjecture**
- **The S duality conjecture**
- **The Maldacena conjecture**

Other conjectures which are believed by many string theorists include,

- **Uniqueness of the non-perturbative ground state conjecture.** There is a unique ground state in string theory, which is the solution of some dynamical problem, such as minimization of some potential.
- **Empirical adequacy conjecture.** That unique string theory ground state leads uniquely to a prediction that the world has $3 + 1$ large dimensions, in which supersymmetry is broken, leading to a phenomenology in agreement with all observations³³.

For there to be a dynamical mechanism to find a unique background, all the different backgrounds must be part of the same theory. So this requires:

- **String theory unification conjecture.** The different background dependent theories are actually expansions around different classical solutions of a single, unified, background independent string theory, and this theory has a connected configuration space.

spacetime must be asymptotically AdS . This is perhaps an attractive proposal, but it itself depends on a positive resolution of some of the open issues discussed above.

³²Although there are suggestive results in a matrix form of string theory that this might become possible[167].

³³Alternatively, if the world is found experimentally to have more than $3 + 1$ uncompactified dimensions, as in the large extra dimension or Randall-Sundrum models[168], this will be the unique prediction of the unique string theory ground state.

To support this conjecture a number of additional conjectures can be made. These generally depend on the S duality conjecture. Indeed, if that conjecture is true, then it can be argued that if one takes together all the existing S and T dualities that all 5 of the distinct string theories in flat ten dimensional spacetime are different descriptions of a single theory.

Additional arguments suggest that this unified theory also has backgrounds which are 11 dimensional. This is because 10 dimensional string backgrounds have in addition to the metric a scalar field. There is some evidence that this scalar field acts like a radius of an additional compactified dimension[169, 170]. While there is apparently no consistent string theory in 11 dimensions, there is a supersymmetric theory of gravity, called 11 dimensional supergravity[171].

There have then been discovered evidence for duality transformations that, at least approximately, relate certain features of 10 dimensional string theories to 11 dimensional supergravity[169, 170]. There is also another interesting theory in 11 dimensions, which is a description of a $2 + 1$ dimensional membrane, moving in 11 dimensional spacetime[172]. This theory also is supersymmetric. However, it is not yet known if it has a consistent quantization. This leads to the

- **\mathcal{M} theory conjecture[169, 170].** There is a background independent formulation of string theory which unifies all the known string theories, 11 dimensional supergravity and the 11 dimensional supermembrane theory.

In some versions of the \mathcal{M} theory conjecture the fundamental degrees of freedom are not strings in a ten dimensional spacetime. They are the three dimensional *membranes*, together with their duals, which are certain six manifolds, called *five-branes*, all existing in 11 dimensions. The idea is that 2 dimensional string worldsheets are approximations to configurations in which one dimension of a membrane curls around one dimension of spacetime, and the radius of that dimension is taken very small. On larger scales, where spacetime seems 10 dimensional, one sees only a two dimensional string.

The beautiful thing about this conjecture is that it offers a possibility of an explanation of the S -duality conjecture in string theory. The different string theories which are conjectured to be related by S duality arise, at least classically, from different ways of wrapping the added dimension of the membrane around the added dimension of space. Thus, if there were an independent definition of \mathcal{M} theory, or at least a consistent definition of the quantum membrane theory in 11 dimensions, one might be able to prove the S duality conjecture. However, at present there is neither, so S duality, as well as \mathcal{M} theory, remain interesting, but so far, unproven, conjectures.

There are of course, various pieces of evidence that have been adduced for these conjectures. Some of them are simply consequences of the symmetries, and can be explained by an understanding of how representations of the 11 dimensional supersymmetry algebra decompose into representations of its 10 dimensional sub-superalgebras. While beautiful mathematically, these hold whether or not there are consistent, quantum theories that realize the dynamics of the objects postulated.

Some interesting developments that led to somewhat stronger results followed a conjecture that the dynamics of \mathcal{M} theory can be formulated as a certain matrix model[66, 67]. This led to some non-trivial calculations of properties that \mathcal{M} theory, were it to exist, would have. However, the matrix model was found to be strongly background dependent. It appears only to exist, or at least be tractable, in a limited set of backgrounds, mainly 11 dimensional Minkowski spacetime and certain low dimensional toriodal compactifications of it. Moreover, even in these cases, it was restricted to a certain limit, associated with light cone coordinates.

Some work inspired by this has gone into attempts to extend this method to the construction of a genuinely background independent matrix model for \mathcal{M} theory[173]-[175], but the results are not considered definitive. In the absence of such a formulation, there is no clear proposal for either the principles or mathematical formulation of \mathcal{M} theory. It remains an interesting conjecture about the existence of a theory we do not so far know how to formulate or construct.

To illustrate the distance between the present results and the set of conjectures which are widely believed by string theorists, we may note that the present evidence is consistent with the following pessimistic conjecture.

- **Minimal string theory conjecture.** String theory only exists as a large number of background dependent theories. Perturbative superstring theory is not defined unambiguously or is not finite past genus two. The various S dualities that have been postulated do not in fact extend beyond the BPS sectors and the different background dependent theories are not isomorphic. There is no connected configuration space and no unified theory that all perturbative string theories represent expansions around. The conformal induction conjecture of Witten is true, but string theory on $AdS^5 \times S^5$ and $N = 4$ supersymmetric Yang Mills theory are inequivalent beyond that correspondence, so that all results to date involving gauge theory/supergravity or gauge theory/string theory correspondences are consequences of either conformal induction or the supersymmetry algebra applied to the BPS states. Nor does string theory give a consistent description of quantum black hole spacetimes, apart from results concerning BPS and near BPS states. Further, there are no perturbative string theories consistent with either a positive cosmological constant, complete supersymmetry breaking or the absence of massless scalar fields.

Of course, most string theorists will be sure that this conjecture is much too pessimistic. I mention it only to emphasize the distance between the picture often presented and assumed in many talks and papers³⁴ on string theory and the actual results to date.

Even if some of this minimal conjecture turns out to be true, the results that are on the table are among the most impressive and far reaching ever achieved in mathematical or theoretical physics. So string theorists have a lot to be proud of. Even if part or all of the

³⁴For example, only one[188] out of fifteen general review papers I consulted mentions that the question of whether superstring perturbation theory is finite and unambiguous to all orders in the genus expansion is unsolved.

minimal conjecture turns out to be true there has been, and will remain a great deal to be learned from string theory, that may very well be relevant for physics.

Of course I do not know which of these conjectures will turn out to be true. The fairest thing to say about string theory is that it is already a very impressive construction of mathematical physics, but that the possibility of its relevance for a theory of nature depends on substantial progress being made on the open conjectures listed here.

7 Other approaches

Before summarizing our findings we should examine some of the other approaches which have been proposed to address the problem of quantum gravity.

7.1 Dynamical triangulation models

These are models in which a quantum spacetime is represented by a simplicial complex. The edge lengths of the elements are fixed, and the degree of freedom is only the way in which a large number of elements are connected together to make a simplicial complex. Each element is assumed to model a region of spacetime on the order of the bare Planck volume.

Two classes of models have been extensively studied, both numerically and analytically. These are the Euclidean dynamical triangulations models[53] and the causal dynamical triangulation models[54].

Results from Euclidean dynamical triangulation models

In 2 dimensions, the dynamical triangulation models are equivalent to random surface models, and also equivalent to Liouville field theory. These models are completely solved, and all methods agree.

In 4 dimensions dynamical triangulation models were studied for several years[53]. The model has two phases and much work was put into determining whether the phase transition between them is first order or second order. Were it second order it would make it possible to show that the low energy limit of the model is general relativity. In fact after much effort it was concluded that the phase transition is first order, so that general relativity is not a low energy limit of the model. This situation is believed by some workers to persist when matter is added, although there are some results to the contrary[194]. Modulo the resolution of this controversy, it is possible that this approach to quantum gravity is ruled out.

Results from Lorentzian dynamical triangulation models

Following this failure, and in part inspired by causal spin foam models[89], a new class of discrete models was investigated, which are dynamical triangulations models of spacetimes with Lorentzian signature. This study has led to very significant results[54]-[56] which include,

a) A solution to the infamous conformal mode problem, demonstrating in detail, that the Lorentzian path integral is well defined[55]. The fluctuations of the conformal model do not, as conjectured by Hawking and collaborators on the basis of semiclassical arguments, cause the path integral to be unbounded. Instead, the fluctuations are controlled by the path integral measure sufficiently so that the path integral remains well defined.

b) In $1+1$ dimensional the critical behavior was found and was discovered to be very different from that of the 2 dimensional Euclidean theory. For example the Hausdorff dimension is 2, rather than 4 as in the case of Euclidean theories. This tells us that naive expectations that the path integral for Lorentzian quantum gravity could be defined by a naive analytic continuation from the Euclidean theory is false.

c) A certain problem, relevant for making a background independent form of string theory was, surprisingly, solved[56]. This arose from a certain approach to string theory, which was however found to fail if the dimension of spacetime was above one. This is called the $c = 1$ problem. Ambjorn, Loll and collaborators have found that this problem is solved when the strings are modeled by their version of Lorentzian dynamical triangulations. Their results show that the theory exists in higher dimensions and that there is a phase transition when one goes above one dimensional. Further, above one dimension, the effective dimension of the string is three in the low energy limit (the Hausdorff dimension). This may be considered to be evidence that a theory of membranes may be relevant for a background independent form of string theory.

7.2 Regge calculus models

These are discrete models of quantum spacetime in which a spacetime history is represented as simplicial triangulation with varying edge lengths. Rather than varying the triangulation with fixed edge lengths, as in the dynamical triangulation models, the triangulation is considered fixed and the edge lengths are varied. This was one of the first models of quantum spacetime to be studied, and it continues to be studied today.

In three spacetime dimensions the model was constructed many years ago by Regge and Ponzano[195]. Although its significance was not appreciated for some years, it was in fact the first example known of a topological field theory, and it remains a paradigmatic example of such a model. Its quantum deformation (where the deformation parameter is, as in loop quantum gravity, inversely related to the cosmological constant) is rigorously defined and yields non-trivial invariants of three manifolds, knots and graphs.

In 4 spacetime dimensions the model has two phases, but, as in the dynamical triangulation case, the transition between them appears to be first order, so that no continuum limit is found[196].

7.3 Causal set models

This is an approach to quantum gravity based on a few simple observations about the role of causal structure in lorentzian geometry. The causal relations among events in a lorentzian

constitute a partial ordering of the events. Given the causal relations among the events of a spacetime, the metric of that spacetime can be reconstructed modulo a local conformal factor and modulo spacetime diffeomorphisms. This implies that causal structure plus a volume element together have exactly the right amount of physical information needed to reconstruct the diffeomorphism equivalence class of a spacetime.

This suggests the following two hypotheses:

- **Weak causal set hypothesis** A quantum spacetime history provides a list of events \mathcal{E} , with a partial order, \mathcal{P} , representing their causal relations. When that quantum spacetime has a semiclassical description in terms of a manifold \mathcal{M} and a diffeomorphism equivalence class of Lorentzian metrics g_{ab} , then 1) the events of \mathcal{P} (or coarse grained sets of them) can be imbedded in \mathcal{M} , 2) the causal structure of \mathcal{P} can be imbedded, modulo some method of coarse graining, in that of g_{ab} and 3) the volume measured by $\sqrt{\det(g)}$ counts the number of events in each region of \mathcal{M} given by the embedding.
- **Strong causal set hypothesis.** At the most fundamental level, a quantum spacetime history consists of nothing but a discrete set of events \mathcal{E} together with their causal relations \mathcal{P} .

The weak causal set hypotheses has been proposed in connection with causal spin foam models[90], as the causal evolution of a spin foam (defined in [89]) gives rise to a discrete set of events with causal relations. The weak causal set hypothesis may then become a tool to be used in the derivation of the low energy limit of a causal spin foam.

The strong causal set hypothesis was previously proposed by Sorkin and collaborators[52] and has been under development since. Recent results take up the proposal in [93] that directed percolation may play a role in the low energy limit of quantum gravity, in order to propose a dynamics for causal set models based on percolation[95].

The main problem the strong causal set hypothesis has to solve is to give a dynamics for causal sets such that it is natural that 3 + 1 dimensional spacetimes emerge at low energies. Large, randomly generated causal sets are known not to resemble the causal structure of any low dimensional manifold. There is some evidence that the recently proposed directed percolation dynamics have a continuum limit that may correspond to a low dimensional geometry.

Another issue to be resolved is that the matter degrees of freedom must also be derived from the causal set. The problem of course is that the fundamental structure is postulated to be so simple, that essentially every feature of our world besides the fact that there are causal relations must be deduced dynamically from the low energy limit of the theory.

Having said this, the causal set program can claim one success, which is a correct prediction of the order of magnitude of the cosmological constant[199]. This is quite striking, given that no other approach has so far anything convincing to say on this crucial problem.

7.4 Twistor theory [200]

This program of research also takes the causal structure of spacetime as more primary than the metric structure. It is based on the construction of a “dual space” to a lorentzian manifold, called twistor space, consisting of all the null lines (or planes in the complexified case.) The causal relations of the manifold are translated into topological relations among submanifolds of twistor space. Twistor theory has been very successful in the context of classical general relativity and field theory, where it has led to important results. Characteristic of these results is that field equations on a spacetime are translated into conditions of complex analyticity on the dual twistor space. For example the self-dual Einstein equations have been solved in closed form in terms of the complex deformations of twistor space. The problem of translating the full Einstein equations into twistor space remains open, but it is still being pursued. There are also related results showing that the structure of general relativistic spacetimes can be expressed in terms of the null rays.

With regard to quantum gravity, twistor theorists, led by Roger Penrose, hypothesize that the structure of twistor space should be translated into quantum theory. This has yet to be carried out fully, but there are intriguing results involving the quantization of fields in twistor space.

Twistor theory is closely tied to loop quantum gravity, in that the same simplification of the Einstein’s equations in terms of the properties of self-dual connections and self-dual two forms plays an essential role in both programs. There are also suggestions that twistor theory is relevant for supergravity and string theory[201].

7.5 Non-commutative geometry

This is a program, proposed originally by Connes[197], which has had much recent influence in quantum gravity and string theory. The basic idea of the original program is to characterize a Euclidean manifold in physical, diffeomorphism invariant observables, in terms of the spectrum of the Dirac operator on the manifold. Connes then showed that there are structures which can be characterized by operator algebras that satisfy certain axioms satisfied by the Dirac operator on a manifold, that are, however, not constructed as manifolds from sets of points. This gives rise to a generalization of differential geometry, called non-commutative geometry. Connes proposed that the standard model of particle physics can be understood elegantly in terms of such a non-commutative geometry.

Because of the use of operator algebras, it is natural to associate non-commutative geometry with quantum geometry. However, it should be cautioned that in some applications, non-commutative geometry is classical, in the sense that the physical $\hbar = 0$, the deformation parameter which signals that the non-commutative manifold is not an ordinary manifold, is then not identified with \hbar .

Nevertheless, there are also proposals for identifying the deformation parameter with \hbar , so that non-commutative geometry becomes a genuine model of quantum geometry³⁵.

³⁵But see [198], for a recent no go theorem on a certain approach to non-commutative geometry as quantum

Apart from its intrinsic study, non commutative geometry has had an influence on string theory, as there are classes of string backgrounds that involve non-commutative ($\hbar = 0$) geometries in their construction. In such applications non-commutative manifolds are sometimes characterized by saying that the coordinates have a non-commutative algebra. This is useful, but it should be mentioned that it is somewhat against the spirit of Connes' original approach, which was formulated in coordinate free, language in terms of diffeomorphism invariants.

The quantum geometry discovered in loop quantum gravity is also (slightly) non commutative, in that operators that measure the areas and volumes of regions of a spatial manifold, strictly speaking fail to commute, although the lack of commutivity is only evident in their action on a small set of states[122].

7.6 Condensed matter physics inspired models

Recently several condensed matter physicists have proposed that quantum spacetime may be modeled in terms of an ordinary quantum statistical system such as a fermi liquid[202, 203, 204]. The idea is that even though such a system is defined with respect to a fixed background metric, and is generally even formulated as a non-relativistic system, there exist phases in which the spectrum of low energy excitations resembles that of massless particles in Minkowski spacetime. In some cases, excitations of different spin have the same propagation velocity, so that the low energy physics may be described to some approximation in terms of a relativistic field theory. Thus, it has been proposed that perhaps the experimental success of special relativity is due to the universe being in such a low temperature phase, of a system that is fundamentally non-relativistic. It is further postulated that general relativity may to some approximation be a manifestation of the dependence of the apparent "speed of light" in these systems on parameters such as density and temperature.

This program, needless to say, challenges the basic assumptions that underlie both special and general relativity. To succeed, it must show that the excitations of a non-relativistic condensed matter system really can be described in terms of relativistic fields. It must explain the emergence at low energies of gauge symmetries and diffeomorphism invariance. Furthermore, such a program is very vulnerable to falsification, as it most likely predicts modifications of the energy-momentum relations in the context of the preferred frame scenario A, discussed above. As discussed there, there are already quite strict limitations on such theories.

At the same time, such studies may be useful as they may shed some light on how quantum critical phenomena may play a role in the emergence of classical spacetime and quantum fields in the low energy limit of a spin foam model.

geometry.

8 How well do the theories answer the questions?

Let us now summarize how well the theories do answering the questions. The status of each of the 24 questions we asked, in string theory and loop quantum gravity, is summarized in Table 1. (The other programs answer each one or a few of the questions, but so far do not address as many as string theory and loop quantum gravity.)

Let us take the questions in turn, beginning with the questions about quantum gravity.

8.1 Quantum gravity questions

Loop quantum gravity gives positive and specific answers to each of the first ten questions concerning quantum gravity. Specifically, so far, for the case of nonzero Λ every question is answered positively, including the existence of a good low energy limit. And, so far, no modification of either the principles of general relativity or quantum theory appears to be required for the existence of a good quantum theory of gravity.

With regard to the second and third question, a complete physical picture of quantum geometry is provided by the theory that differs in striking and specific ways from the classical theory of spacetime geometry.

With regard to the fourth question, there is a microscopic description of black hole horizons, which reproduces and explains the Bekenstein entropy in terms of conventional coarse grained description of microstates, in this case microstates of the horizon degrees of freedom. Furthermore, calculations lead to a derivation of the Hawking spectra, with computable corrections.

There is no problem with a positive cosmological constant, in fact this is the best case for the theory as here we have simultaneously a microscopic and semiclassical description in terms of a single exact solution to the quantum constraints. Further, the temperature and entropy of deSitter spacetime are understood.

Recent results show that cosmological singularities are removed. There are no results yet concerning black hole singularities.

The theory is fully background independent.

The present indications are that there are Planck scale modifications in the realization of global lorentz invariance, leading to predictions for physical effects that may be observable in the present and certainly will be testable in the near future.

With respect to question 11, the situation is not satisfactory, in that no calculations of the scattering of gravitons past the classical approximation have been yet carried out in loop quantum gravity. It can be hoped that progress can be made soon, at least in the case of non-zero cosmological constant.

Thus, we may summarize by saying that so far loop quantum gravity provides an answer to the first ten questions concerning quantum gravity. While the 11'th question remains unresolved, there is now work in progress which has a realistic chance of addressing it.

Next, we consider how well string theory answers the quantum gravity questions.

Table 1: Summary of results. A=solved. B=partial results, or solved in some cases, open in others. C=in progress using known methods. ?= requires the invention of new, so far unknown methods. -=makes no claims to solve.

Question	String theory	Loop Quantum Gravity
Quantum Gravity		
1. <i>GR and QM true or need modification?</i>	A	A
2. <i>Describes nature at all scales?</i>	B	A
3. <i>Describes quantum spacetime geometry?</i>	B	A
4. <i>BH entropy and temperature explained?</i>	B	A
5. <i>Allows $\Lambda > 0$?</i>	?	A
6. <i>Resolves singularities of GR?</i>	B	B
7. <i>Background independent?</i>	?	A
8. <i>New predictions testable now?</i>	?	B
9. <i>GR as low energy limit?</i>	A	B
10. <i>Lorentz invariance kept or broken?</i>	A	B
11. <i>Sensible graviton scattering?</i>	B	C
Cosmology		
1. <i>Explains initial conditions?</i>	?	C
2. <i>Explains inflation?</i>	C	C
3. <i>Does time continue before big bang?</i>	?	A
4. <i>Explains the dark matter and energy?</i>	?	?
5. <i>Yields transplankian predictions?</i>	C	C
Unification of forces		
1. <i>Unifies all interactions?</i>	A	-
2. <i>Explains $SU(3) \times SU(2) \times U(1)$ and fermion reps?</i>	?	-
3. <i>Explains hierarchies of scales?</i>	?	-
4. <i>Explains values of standard model parameters?</i>	?	-
5. <i>Unique consistent theory?</i>	?	-
6. <i>Unique predictions for doable experiments?</i>	?	B
Foundational questions		
1. <i>Resolves problem of time in QC?</i>	?	C
2. <i>Resolves puzzles of quantum cosmology?</i>	?	C

String theory offers a possible answer to the first question of how gravitation and quantum theory are unified, which does not require that the principles of general relativity and quantum theory be exactly compatible. There is also evidence that it may provide a solution to question 11, in the form of superstring perturbation theory, so long as that theory can be shown to be finite and unambiguous to all orders in the genus expansion. These are both strong successes of the theory.

In certain restricted cases, string theory does provide an answer to question 2. These are BPS states, where the existence of T and S duality transformation allows quantum geometry for scales shorter than the string scale to be described in a dual theory in terms of scales larger than the string scale. In some cases, no deviation from the classical picture in which spacetime is continuous and smooth, are seen. In other cases, it appears that the classical picture of spacetime geometry becomes replaced by a non-commutative, but still classical, (in the sense of $\hbar \rightarrow 0$), spacetime geometry.

It is not known whether these results extend to all states and solutions of a string theory, beyond the restricted set of BPS states where calculations and duality transformations can be explicitly carried out.

With regard to question 4, there are striking results in the case of systems with the same quantum numbers as extremal and near extremal black holes. These results extend even to the computation of grey body factors, in exact agreement with the semiclassical results. These suggest, but do not show, that string theory may in the future give a detailed microscopic description of quantum black holes. It is not known presently whether these results extend to all black holes. If they do not it may be that these results are accidental, in that they are forced by the supersymmetry algebra that, in the case of BPS and near BPS states, strongly constrains the spectrum and degeneracies of the Hamiltonian.

String theory so far does not appear to incorporate deSitter spacetime as a consistent background, and hence has trouble with a positive cosmological constant.

There are results that indicate that various kinds of singularities of classical general relativity can be removed by string theory. These however do not so far include either cosmological or black hole singularities.

String theory is not background independent, and as such offers nothing new concerning question 9³⁶.

Finally, string theory gives a theory of graviton scattering which is known to be unambiguous and finite to second order in perturbation theory.

8.2 Cosmological questions

Next, we come to the cosmological questions.

³⁶The few attempts to construct truly background independent formulations of string theory have, in my view, been promising, but have not generated so far strong interest. Besides the proposal in [174, 175], there are a few other approaches, for example an approach based on an 11 dimensional Chern-Simons theory[181]. While interesting, this proposal faces the serious difficulty that 11d Chern-Simons theory has many local degrees of freedom whose dynamics and canonical structure are very tricky to untangle[182, 183, 27]

Recent results in loop quantum gravity shows some promise of answering the first three of the questions. In particular, there are results that indicate that cosmological singularities are eliminated and the evolution of the quantum universe continues in time through the singularities of classical *FRW* universes[117]. Calculations concerning transplankian effects are in progress. But loop quantum gravity has nothing to say about the dark matter and energy.

String theory has so far little to say definitively about cosmological questions. There are a number of ideas and models under development to address those questions, using higher dimensions or the idea that the universe lives on a brane in a higher dimensional manifold. Some of these ideas are closely tied to string theory, others are not. It is not yet clear whether this direction will lead to experimental tests of string theory, but it is a possibility. Very recently there have begun attempts to compute transplankian effects using string theory.

8.3 Questions concerning unification

Loop quantum gravity has, so far, nothing to say about the question of unification. While there are some speculations in this direction³⁷, and some work on background independent approaches to string theory, none of this has led to any definitive progress on the questions asked. However, as mentioned, it does appear that loop quantum gravity makes predictions for experiments that test lorentz invariance at high energies.

String theory was invented to be a unified theory of all the interactions and its main strength remains the fact that it gives a beautiful and, to many, compelling solution to the first question. This is a great success, such theories do not grow on trees.

At the same time, in a certain sense they do, as there turn out to be an infinite number of background dependent string theories, all of which provide consistent perturbative unifications of gauge theories with gravity, coupled to a variety of matter fields including fermions, at least through genus two in perturbation theory.

As such, string theory so far provides no answer to the other questions concerning unification. There is so far no known stable string background that predicts all the observed features of particle physics phenomenology, or resolves any of the open questions concerning the standard model of particle physics. String theory makes so far three clear predictions: 1) supersymmetry should be found at some scale between a Tev and the Planck scale. 2) at some scale some evidence for additional dimensions must be found, 3) exact lorentz invariance should be preserved.

All three of these are the object of current experimental programs. One should note that supersymmetry and evidence of additional dimensions or degrees of freedom need not be discovered in near future experiments, as the only absolute prediction is they must be discovered somewhere below the Planck scale. Nor would the discovery of Tev scale supersymmetry prove the correctness of string theory, as there are ordinary supersymmetric field

³⁷Two proposals for unification within loop quantum gravity are described in [207] and [208] showing that this is a possibility that merits further exploration.

theories which are extensions of the standard model. Nor would the discovery of supersymmetry rule out loop quantum gravity, as all the main results of loop quantum gravity can be extended to supergravity, at least through $N = 2$.

It is then evident that present and near future tests for the presence of Planck scale corrections to the energy momentum relations offer both the best way to falsify string theory and the best way to distinguish experimentally between string theory and loop quantum gravity.

Further, given the existence of an infinite number of string theory backgrounds, in the absence of a complete non-perturbative formulation of string theory, it seems that there is not on the horizon any way to strongly confirm unique predictions of string theory, as opposed to predictions of supersymmetric grand unified theories. For example, it is compatible with all known results that, if there is any string theory background consistent with all known experiments, there are a large and perhaps infinite number of such models, that would give different predictions for phenomenology at TeV and higher scales. In this case string theory would still fail to be predictive, beyond the general prediction that Lorentz invariance should be realized linearly at all scales.

Finally, it should be emphasized that so far neither string theory nor loop quantum gravity have much new to offer to resolve the questions of the large hierarchy of scales. They are both compatible with various mechanisms proposed to resolve the gauge hierarchy problems in grand unified theories, as there are versions of each that incorporate the basic features of (possibly supersymmetric) grand unified theories. Both theories also appear to be compatible with known field theoretic mechanisms for the spontaneous breaking of supersymmetry. Neither theory has so far anything to offer to explain why the cosmological constant is so small, although at least loop quantum gravity appears to have no problem with the apparently observed fact that the sign is positive.

8.4 Foundational questions

This leaves the last two questions, concerning the problem of time and the problem of quantum theory in a closed universe. Since loop quantum gravity provides a precisely defined example of a quantum theory of gravity and cosmology these problems may now be investigated with a precision that was not previously possible. The result, to summarize a lot of work, is that most of the so far proposed solutions to the problem of time in quantum cosmology can be formulated in detail and tested in the context of loop quantum gravity. There is presently a lively debate concerning this issue, so I will not try here to predict the outcome, except to say that there appears to be no reason to believe the problem is no more difficult in principle than that in the full classical theory, with cosmological boundary conditions. That is, if one sticks strictly to discussing physical, gauge invariant observables, and is careful to ask only physical questions, than the different notions of time which have proved useful in the classical theory can be constructed and represented in the quantum theory.

With regard to the question of the formulation of a measurement theory for quantum

cosmology, when the observer is part of the universe, much the same situation obtains. The different possible solutions which have been proposed can be expressed exactly in the Hilbert space of solutions to the Hamiltonian constraint of loop quantum gravity.

While this is an area of lively debate, it may be noted that a new kind of solution to the problem of providing a measurement theory to quantum cosmology has been formulated by people working in loop quantum gravity[68, 178, 177, 179, 180]. This is called *relational quantum cosmology* and the basic physical ideas are due to Crane, Rovelli and Markopoulou. A mathematical structure for a generalization of quantum theory that appears compatible with their ideas has been proposed by Butterfield and Isham[180].

The basic ideas of relational quantum cosmology are described in an appendix.

9 Conclusions

The first thing that must be said is that if we compare what we know now about quantum gravity to what we knew twenty years ago, it is clear that there has been enormous progress. This is due to an enormous effort by a large number of people, who choose to dedicate their time and, in many cases, their careers to pursue this very risky venture, when they might have found success more quickly elsewhere. It is impossible to look at the list of results in these and other approaches to quantum gravity over the last twenty years and not feel in awe of the enormous talent, intelligence and hard work that people have contributed.

Second, both string theory and loop quantum gravity are very much alive as research programs with a significant chance of uncovering new laws of nature. Each has achieved much more than prudent experts would have bet was possible twenty years ago. Certainly more progress has been made on quantum gravity than I expected to see in my lifetime. So it is clear that both loop quantum gravity and string theory should continue to be pursued vigorously. Each deserves significant support from the physics and academic communities.

At the same time, my own conclusion after the exercise of writing this review is that the two theories are in very different situations. To explain this impression I would like to propose a list of what would need to be done in each case to finish the theoretical program and bring each theory to the point where it could be compared with real, doable, experiments. Any such list requires a certain amount of speculation and guess work, and I am sure that different people would produce, if asked, different lists. But it is interesting nonetheless to get an idea of what remains to do in each case.

9.1 What remains to be done in loop quantum gravity?

1. For general Λ develop the method of coherent states to discover the conditions for a loop quantum gravity theory to have a consistent low energy limit, develop perturbation theory around it and test whether it is consistent to all orders.
2. For the case $\Lambda \neq 0$ develop perturbation theory for excitations around the Kodama state and test whether it is consistent and sensible order by order.

For general Λ , continue the development of renormalization group methods in the context of spin foams, to discover which loop quantum gravity theories have good low energy limits.

3. Develop a method in loop quantum cosmology to predict transplankian effects in cosmology and, when they are accessible to test, compare them to precision measurements of *CMB* spectra.
4. Refine the existing calculations that predict modified energy-momentum relations, to determine whether or not the theory makes unique predictions for the parameters $\alpha, \beta \dots$ in the modified energy-momentum relations (1), for the different particle species. Resolve the question of whether Lorentz invariance is realized exactly, broken or realized non-linearly in the low energy limit of loop quantum gravity.
5. Develop a dynamical formulation of spacetimes with horizons in order to understand dynamically the connection between the discreteness of area and the quasi normal mode spectrum discovered by Dreyer.
6. Work out the details of a version of the holographic principle in the context of relational quantum cosmology, making use of the fact that the Bekenstein bound is realized naturally in constrained topological field theories.

9.2 What remains to be done in string theory?

1. Resolve the problem of the existence, uniqueness and consistency of superstring perturbation theory past genus two.
2. Demonstrate the existence of at least one string perturbation theory consistent with all the features of our universe, including completely broken supersymmetry, a positive cosmological constant and the absence of massless or light fundamental scalar fields.
3. Discover whether this theory, if it exists, is unique and, if so, whether any predictions can be made for near term experiments.
4. Related to the foregoing, understand how and why supersymmetry, if present at all, is spontaneously broken.
5. Discover whether the *S* duality conjecture is true or false.
6. Discover whether the Maldacena conjecture is true or false.
7. Find a background independent formulation of string or \mathcal{M} theory. Find the classical solutions to this theory and show that the different perturbative string theories do arise as expansions around them.

8. Formulate a principle that picks out a unique perturbative string theory which may be our universe. Explain why this principle picks a universe with all the features of ours, including having $3 + 1$ ordinary, large dimensions, broken supersymmetry, no massless scalars, and whose low energy phenomenology is given by the standard model.
9. Follow this with calculations of the parameters of the standard model of particles physics and, perhaps, cosmology.
10. Make unique predictions for phenomena that are beyond the predictions of the standard model, but accessible to present or near term experiments.
11. Fully develop a version of the holographic principle in string theory, either by proving the Maldacena conjecture or else by finding an alternative formulation. Show that it applies also to cosmological spacetimes with horizons.
12. Find methods to study general black hole spacetimes in string theory.
13. Develop an approach to cosmological singularities in string theory. Then develop a method to extract predictions for transplankian effects and compare them to future *CMB* observations.
14. Finally, give a simple set of postulates for string theory from which all the results relevant for the description of nature may be derived.

Different people might propose different items for such a list. In mine there is a clear difference which emerges, which brings out the differences in the two research programs.

This is due perhaps to the fact that string theory is a far more ambitious program than loop quantum gravity. String theory is perhaps best understood to be a research program in search of new postulates for fundamental physics, whereas loop quantum gravity is based on the combination of the relatively well understood principles of quantum theory and general relativity. As a result, loop quantum gravity is perhaps less ambitious, but because of this it appears to be significantly closer to completion. After a long period of development during which results have accumulated, the claims that could be made for loop quantum gravity have steadily strengthened. At this point the theory is well enough understood that it is possible to formulate a program to bring it to completion and experimental test over the next several years, using only known ideas and methods.

What remains to be done on the theory side requires mostly the application of standard methods such as perturbation theory and renormalization group techniques to well defined theories. Regarding cosmology there is a research program under development, using standard methods, that is likely to result in predictions for transplankian phenomena, that may be testable. For $\Lambda \neq 0$ there are good indications that the difficult problem of showing that general relativity is the low energy limit may be solved, and perturbation theory is presently under development. Even for $\Lambda = 0$, where the Kodama state does not guarantee

the existence of a good low energy limit, there are techniques under development which, while computationally challenging, should allow increasing control over the low energy limit.

Meanwhile, a set of experiments that may allow the theory to be tested have been identified and calculations are underway to sharpen the predictions for them. These require only the application of standard methods of theoretical physics.

No new principles are required because loop quantum gravity I takes as its postulates only the principles of general relativity and quantum theory, and these have the great advantage of being well confirmed experimentally. What the results tell us is that, when due attention is paid to issues of how to incorporate the gauge invariances of general relativity in a quantum field theory, there appears no obstacle to the joint application of these well established sets of principles. What remains may be no more than the straightforward working out of the consequences for experiment.

Loop quantum gravity I may fail. For example, the predictions of quantum general relativity may turn out to disagree with experiment. The good news is that this may occur within the next ten years. Even if it does, loop quantum gravity II offers a conceptual and mathematical framework for a large class of quantum theories of space and time. It is then not a specific theory, it is more analogous to lattice gauge theory in being a general technique to investigate theories with certain kinds of symmetries, in this case diffeomorphism invariance. It can even be said that loop quantum gravity II offers a framework for the ambition motivating string theory to be realized that is not hampered by background dependence and is based on a complete and exact unification of quantum theory with the basic principle that space and time are fully dynamical and background independent. But even loop quantum gravity II may turn out to make generic predictions that disagree with experiments to be carried out in the next decade.

By comparison, the situation of string theory is much less clear, at least for the near future. One problem is that several of the steps on the list remain unsolved, after many attempts over many years. It is possible then that these will require the discovery of new, presently unknown, principles and, quite possibly, also substantial mathematical and technical innovations. As what is needed goes significantly beyond what is known, it seems not easy to predict when, or over what path, string theory may be able to take the steps necessary to become a completed physical theory.

Nevertheless, progress is continuing. One question on which there are new approaches, if not yet a solution, is the second, that of finding a string perturbation theory that is not ruled out as realistic by some observed feature of the world³⁸. It is possible that this may be achieved by continuing to construct models with existing methods. At the same time, it is consistent with present results to conjecture, as does Banks[184], that there are no consistent string perturbation theories that admit a positive cosmological constant, as presently observed, or that have completely broken supersymmetry with no massless scalar

³⁸Very recently proposals have been made for string vacua that have positive cosmological constants and break supersymmetry[192]. These theories are all unstable and, it is argued, must decay. There are indications that some of these theories may have decay times long compared to the present Hubble scale, but it is not yet clear that all decay channels have been understood.

fields. At the very least, it appears possible that string theories with these characteristics, if they exist at all, may require the introduction of new methods.

As an indication of this situation, it is interesting that recently a few string theorists have been resorting to mention of the anthropic principle in order to resolve the problem of connecting the theory with observation. Recently Susskind[193] and others have proposed that there will be a huge number of consistent non-supersymmetric string vacua (although still not one stable non-supersymmetric string vacua has so far been written down) and that there will be no selection mechanism to pick out a preferred one except the anthropic principle³⁹. This appears to represent quite a change of perspective from claims that string theory in the end would lead to a unique recovery of standard model physics and testable predictions for observations beyond the standard model. This is another indication that string theory is a search for new principles.

These difference are certainly due to the fact that the two programs have very different ambitions. String theory began as a search for a conjectured unique theory that would unify all of nature. In spite of the fact that at the background dependent level string theory is far from unique, to a large extent this is still the prime motivation of the program. To realize this hope, string theory relies on several mathematical conjectures which remain unproven, in spite of intense effort, and several physical hypotheses, which may turn out to be right or wrong. While the idea of duality, that gauge and other degrees of freedom may be described in terms of stringlike excitations, is attractive, the cost of realizing it as a fundamental, rather than an effective theory, appears high. Either there are or are not extra dimensions, and supersymmetry is either part of the laws of nature or not. In the end only experiment can tell, but there appears to be no near term experimental program which could falsify these hypotheses. What is so frustrating about string theory is that it could easily be wrong, in whole or in part, but there appear to be few realistic ways to find out. The only possibility I know of for near term falsification are the tests of lorentz invariance at high energies.

On the other hand loop quantum gravity makes much less radical assumptions, and instead investigates the question of *how* to fully reconcile the basic physical ideas and principles that underlie relativity and quantum theory. While still incomplete, loop quantum gravity has clearly succeeded in partly solving this problem, resulting in several novel physical predictions, and it is the only research program that has done so. String theory has so far largely ignored the problems loop quantum gravity has taken on, and to a large extent solved.

If string theory is right then sooner or later it will have to attack the problem of how to have a fully consistent background independent quantum theory of space and time. It will then have to begin to address the issues that loop quantum gravity has already gone a long way towards solving.

From the point of view of loop quantum gravity II, supersymmetry and higher dimensions

³⁹For a detailed discussion of exactly what kinds of “anthropic reasoning” do and don’t lead to theories that satisfy the basic test of a scientific theory that it be falsifiable, see [64]. As described there in detail, most versions of the anthropic principle fail to be falsifiable. An example of a genuinely falsifiable theory (which incidentally has so far survived attempts to falsify it) is cosmological natural selection, described also in [64].

can be easily incorporated and there is also good reason to believe that strings may emerge as an effective description at a scale below the Planck scale[173]. Thus, from the point of view of loop quantum gravity, if experiment shows that the world is supersymmetric or higher dimensional, there need be no obstacle to describing it in background independent terms. Thus it remains a possibility that in the end string theory and loop quantum gravity will come together because the methods and results of loop quantum gravity will turn out to be indispensable for the solution of the problem of making a background independent form of string theory.

The most important conclusion of this survey is that there is now a realistic chance that experiment may over the next ten years be able to distinguish between the predictions of different quantum theories of gravity, including string theory and loop quantum gravity. Given this, the first priority of theory must be to anticipate the experiments, by bringing the theories to the point where they make clean predictions that may allow them to be falsified.

ACKNOWLEDGEMENTS

This review started as an essay commissioned in honour of John Wheeler, and although it was not finished in time for that I would like to thank the editors of the Wheeler volume and the Templeton foundation for the invitation to write it. I also have to thank many people for discussions and correspondence over the last several years that were helpful to its preparation. These include Giovanni Amelino Camelia, Tom Banks, John Brodie, Shyamolie Chaudhuri, Michael Douglas, Willy Fischler, Michael Green, Brian Greene, David Gross, Murat Gunyadin, Gary Horowitz, Chris Hull, Chris Isham, Clifford Johnson, Renata Kallosh, Jurek Kowalski-Glikman, Frederic Leblond, Juan Maldacena, David Mateos, Djorge Minic, Rob Myers, Herman Nicolai, Amanda Peet, Joe Polchinski, Lisa Randall, Konstatin Saviddy, Steve Shenker, Kelle Stelle, Andy Strominger, Raman Sundrum, Lenny Susskind, Thomas Thiemann, Arkady Tseytlin and Edward Witten. I am especially grateful to Eric D'Howker, Clifford Johnson, Gordy Kane, Juan Maldacena, Rob Myers, John Schwarz, and Arkady Tseytlin for taking the time to answer questions that came up while writing this review, to Seth Major for very helpful comments on the text and to Fotini Markopoulou for very helpful critical advice. I also must thank my collaborators over the years who have taught me most of what I know about quantum gravity, Stephon Alexander, Matthias Arnsdorf, Abhay Ashtekar, Roumen Borissov, Louis Crane, John Dell, Ted Jacobson, Laurent Friedel, Yi Ling, Seth Major, Joao Magueijo, Fotini Markopoulou, Carlo Rovelli and Chopin Soo. I am grateful as well to the NSF and the Phillips Foundation for their very generous support which made my own work possible. Finally I would like to thank my friends and colleagues at Perimeter Institute for the open, critical, but friendly atmosphere which was so helpful in preparing this review and especially to Frederic Leblond for an invitation to present this work at the PI string seminar and for many helpful and critical remarks there.

APPENDIX: Relational Quantum Cosmology

For the interested reader, I summarize here the basic ideas of relational quantum cosmology.

- Crane: Hilbert spaces are associated with boundaries that split the universe into parts. By the relationship of GR to TQFT these will be described in terms of *finite dimensional state spaces*[68].
- Rovelli: The Hilbert space describes the information one part of the universe has about another part[177].
- Markopoulou: Different Hilbert spaces are associated with local observers inside the universe and describe information coming from their past light cones. Quantum cosmology without the wave function of the universe[179].
- Butterfield and Isham: The right mathematics for relational quantum theory is topos theory[180].

Reduced to a slogan, relational quantum cosmology maintains that, “Many quantum states to describe one universe, not one state describing many universes.”

In fact relational quantum cosmology is closely connected to the holographic principle. Indeed, its original formulation, by Crane, preceded 't Hooft's papers on the holographic principle, and should probably be considered the first statement of the principle.

One version of the holographic principle connected with relational quantum cosmology has been proposed, which may be called the *weak holographic principle*. It may be summarized as saying that[71]

- A surface in space is a channel through which quantum information flows. All measurements are made on such surfaces. Each surface has associated to it a Hilbert space that contains the possible outcomes of measurements made on that surface.
- The area of the surface is another name for its capacity as a channel of quantum information. The log of the dimension of the Hilbert space of each surface is hence taken to be a definition of its area. In this way geometry is reduced ultimately to information theory.
- This is the basis of a measurement theory for spatially closed causal spin foam

In fact, as conjectured by Crane in the paper that stimulated these developments[68], in loop quantum gravity there are Hilbert spaces associated with boundaries and they are constructed from Chern-Simons theory, which is a topological field theory. Moreover they are, as Crane conjectured, finite dimensional and they do automatically implement Bekenstein's bound. Thus, while there remains work to do to fully formulate a relational quantum cosmological theory, it may be said that loop quantum gravity does have some features suggested by relational quantum theory.

Relational quantum theory has also been explored in a few papers by string theorists, particularly Banks and Fischler[185]. They propose that when the cosmological constant, $\Lambda > 0$, supersymmetry is necessarily broken and the quantum theory of gravity is described in terms of finite dimensional Hilbert spaces. They then propose an approach that appears to have some elements in common with that of relational quantum cosmology, particularly in the form given by Markopoulou[179].

One consequence of these ideas is that there is, at least when $\Lambda > 0$, no need for Hilbert spaces that appear in the theory to be infinite dimensional. Instead, one can argue that N , the dimension of any local Hilbert space arising in the theory is bounded by $\frac{G\Lambda}{3}$. This bound was conjectured by Banks[184] on the basis of the fact that this is the entropy of deSitter spacetime.

Thus, the conclusion is that new ideas have arisen in loop quantum gravity which have some hope of resolving the problem of time and the problem of quantum cosmology. Further, as a well formulated background independent quantum theory, loop quantum gravity allows older ideas about these problems to be precisely formulated and tested. Meanwhile, while string theory apparently does not offer so far anything new to resolve these problems, it is striking that a few string theorists have put forward proposals that appear to be inspired by the ideas coming from loop quantum gravity.

References

- [1] J. D. Bekenstein, Black Holes And Entropy, Phys. Rev. D 7, 2333 (1973).
- [2] S. W. Hawking, Particle Creation By Black Holes,” Commun. Math. Phys. 43, 199 (1975).
- [3] W. G. Unruh, Phys. Rev. D 14 (1976) 870.
- [4] M. B. Green, J. H. Schwarz and E. Witten, Superstring Theory.” Cambridge University Press (1987);
- [5] J. Polchinski, ”String Theory”, Vol. 1 and 2, Cambridge University Press (1998);
- [6] B. R. Greene, The Elegant Universe,” New York, Norton (1999) 448 p.
- [7] C. Rovelli, Living Rev. Rel. 1 (1998) 1, gr-qc/9710008.
- [8] A. Ashtekar, New perspectives in canonical gravity (Bibliopolis, Naples, 1988); Lectures on non-perturbative canonical gravity, Advanced Series in Astrophysics and Cosmology- Vol. 6 (World Scientific, Singapore, 1991).
- [9] R. Gambini and J. Pullin, Loops, knots, gauge theories and quantum gravity Cambridge University Press, 1996.

- [10] L. Smolin: in *Quantum Gravity and Cosmology*, eds J Perez-Mercader et al, World Scientific, Singapore 1992; The future of spin networks gr-qc/9702030 in the Penrose Festschrift.
- [11] A. Perez, *Spin Foam Models for Quantum Gravity*, Topical Review in Class.Quant.Grav. 20 (2003) R43 gr-qc/0301113.
- [12] A. Sen, On the existence of neutrino zero modes in vacuum spacetime J. Math. Phys. 22 (1981) 1781, Gravity as a spin system Phys. Lett. B11 (1982) 89.
- [13] Abhay Ashtekar, New variables for classical and quantum gravity,” Phys. Rev. Lett. 57(18), 2244-2247 (1986).
- [14] T. Jacobson and L. Smolin, Nucl. Phys. B 299 (1988) 295.
- [15] C. Rovelli and L. Smolin, Knot theory and quantum theory, Phys. Rev. Lett 61(1988)1155; Loop representation of quantum general relativity, Nucl. Phys. B331(1990)80-152.
- [16] R. Gambini and A. Trias, Phys. Rev. D23 (1981) 553, Lett. al Nuovo Cimento 38 (1983) 497; Phys. Rev. Lett. 53 (1984) 2359; Nucl. Phys. B278 (1986) 436; R. Gambini, L. Leal and A. Trias, Phys. Rev. D39 (1989) 3127.
- [17] C. Rovelli and L. Smolin, Spin networks and quantum gravity, gr-qc/9505006, Physical Review D 52 (1995) 5743-5759.
- [18] C. Rovelli and L. Smolin Discreteness of area and volume in quantum gravity, Nuclear Physics B 442 (1995) 593. Erratum: Nucl. Phys. B 456 (1995) 734; L. Smolin, “Finite Diffeomorphism Invariant Observables In Quantum Gravity,” Phys. Rev. D **49**, 4028 (1994) [arXiv:gr-qc/9302011].
- [19] L. Smolin *Three Roads to Quantum Gravity* (Weidenfeld and Nicolson and Basic Books, London and New York,2001)
- [20] <http://superstringtheory.com/>, www.qgravity.org. Many talks and conferences on string theory are available on line, see these web sites, or just search for them!
- [21] Peter Woit, *String Theory: An Evaluation*, physics/0102051.
- [22] Daniel Friedan, *A tentative theory of large distance physics*, hep-th/0204131.
- [23] Y. Ling and L. Smolin, Supersymmetric spin networks and quantum supergravity, Phys. Rev. D61, 044008(2000), hep-th/9904016; Holographic Formulation of Quantum Supergravity, hep-th/0009018, Phys.Rev. D63 (2001) 064010.

- [24] T. Jacobson, New Variables for canonical supergravity, *Class. Quant. Grav.*5(1988)923-935; D. Armand-Ugon, R. Gambini, O. Obregon, J. Pullin, Towards a loop representation for quantum canonical supergravity, hep-th/9508036, *Nucl. Phys. B*460 (1996) 615; L. F. Urrutia Towards a loop representation of connection theories defined over a super-lie algebra, hep-th/9609010; H. Kunitomo and T. Sano The Ashtekar formulation for canonical N=2 supergravity, *Prog. Theor. Phys. suppl.* (1993) 31; Takashi Sano and J. Shiraishi, The Nonperturbative Canonical Quantization of the N=1 Supergravity, *Nucl. Phys. B*410 (1993) 423, hep-th/9211104; The Ashtekar Formalism and WKB Wave Functions of N=1,2 Supergravities, hep-th/9211103; T. Kadoyoshi and S. Nojiri, N=3 and N=4 two form supergravities, *Mod. Phys. Lett. A*12:1165-1174,1997, hep-th/9703149; K. Ezawa, Ashtekar's formulation for N=1, N=2 supergravities as constrained BF theories, *Prog. Theor. Phys.*95:863-882, 1996, hep-th/9511047.
- [25] Yi Ling, "Introduction to supersymmetric spin networks", hep-th/0009020, *J.Math.Phys.* 43 (2002) 154-169
- [26] L. Freidel, K. Krasnov, R. Puzio, "BF Description of Higher-Dimensional Gravity Theories", hep-th/9901069, *Adv.Theor.Math.Phys.* 3 (1999) 1289-1324.
- [27] Y. Ling and L. Smolin, "Eleven dimensional supergravity as a constrained topological field theory," *Nucl. Phys. B* **601**, 191 (2001) [arXiv:hep-th/0003285]; L. Smolin, "Chern-Simons theory in 11 dimensions as a non-perturbative phase of M theory," arXiv:hep-th/9703174.
- [28] G. Amelino-Camelia et al, *Int.J.Mod.Phys.A*12:607-624,1997; G. Amelino-Camelia et al *Nature* 393:763-765,1998; J. Ellis et al, *Astrophys.J.*535:139-151,2000; J. Ellis, N.E. Mavromatos and D. Nanopoulos, *Phys.Rev.D*63:124025,2001; *ibidem astro-ph/0108295*.
- [29] N. Hayashida, K. Honda, N. Inoue et. al. (The AGASA collaboration), *Updated AGASA event list above $4 \times 10^{19} eV$* , astro-ph/0008102, *Astrophys.J.* 522 (1999) 225; M. Takeda, N. Sakaki, K. Honda, (The AGASA collaboration), *Energy determination in the Akeno Giant Air Shower Array experiment*, astro-ph/0209422.
- [30] G. Amelino-Camelia and T. Piran, *Phys.Rev. D*64 (2001) 036005.
- [31] G. Amelino-Camelia, *Nature* 418 (2002) 34.
- [32] J. Lukierski, H. Ruegg, A. Nowicki, and V.N. Tolstoy. *q-deformation of Poincare algebra.* *Phys. Lett. B*, 264,271:331,321, 1991; S. Majid. *Braided groups and algebraic quantum field theories.* *Lett. Math. Phys.*, 22:167 176, 1991; *Braided Momentum Structure of the q-Poincare Group*, hep-th/9210141, *J.Math.Phys.* 34 (1993) 2045
- [33] N.R. Bruno, G. Amelino-Camelia, J. Kowalski-Glikman, *Phys.Lett.B*522:133-138,2001; J. Kowalski-Glikman and S. Nowak, hep-th/0203040; S. Judes, gr-qc/0205067; M. Visser, gr-qc/0205093; S. Judes, M. Visser, gr-qc/0205067; D. V. Ahluwalia and M. Kirchbach, gr-qc/0207004.

- [34] J. Magueijo and L. Smolin, Phys.Rev.Lett. (88) 190403, 2002.
- [35] J. Magueijo and L. Smolin, gr-qc/0207
- [36] Rodolfo Gambini, Jorge Pullin, “Nonstandard optics from quantum spacetime”, Phys.Rev. D59 (1999) 124021, gr-qc/9809038;
- [37] Jorge Alfaro, Hugo A. Morales-Tecotl, Luis F. Urrutia, “Loop quantum gravity and light propagation”, Phys.Rev. D65 (2002) 103509, hep-th/0108061.
- [38] L. Smolin, *Quantum gravity with a positive cosmological constant*, hep-th/0209079.
- [39] Tomasz J. Konopka, Seth A. Major, “Observational Limits on Quantum Geometry Effects”, New J.Phys. 4 (2002) 57. hep-ph/0201184; Ted Jacobson, Stefano Liberati, David Mattingly, “ TeV Astrophysics Constraints on Planck Scale Lorentz Violation”; *Threshold effects and Planck scale Lorentz violation: combined constraints from high energy astrophysics*, hep-ph/0112207.
- [40] Subir Sarkar, “ Possible astrophysical probes of quantum gravity” , Mod.Phys.Lett. A17 (2002) 1025-1036, gr-qc/0204092.
- [41] S.D. Biller et al, *Limits to Quantum Gravity Effects from Observations of TeV Flares in Active Galaxies*, gr-qc/9810044; G. Amelino-Camelia, *Improved limit on quantum-spacetime modifications of Lorentz symmetry from observations of gamma-ray blazars*, gr-qc/0212002.
- [42] G. Amelino-Camelia, John Ellis, N.E. Mavromatos, D.V. Nanopoulos, Subir Sarkar, *Potential Sensitivity of Gamma-Ray Burster Observations to Wave Dispersion in Vacuo*, astro-ph/9712103, Nature 393 (1998) 763-765; J.P. Norris, J.T. Bonnell, G.F. Marani, J.D. Scargle, *GLAST, GRBs, and Quantum Gravity*, astro-ph/9912136.
- [43] T. Jacobson, S. Liberati, D. Mattingly, *Lorentz violation and Crab synchrotron emission: a new constraint far beyond the Planck scale*, astro-ph/0212190.
- [44] Robert C. Myers, Maxim Pospelov, *Experimental Challenges for Quantum Gravity*, p-ph/0301124.
- [45] Reinaldo J. Gleiser, Carlos N. Kozameh, *Astrophysical limits on quantum gravity motivated birefringence*, gr-qc/0102093.
- [46] Richard Lieu, Lloyd W. Hillman, *Stringent limits on the existence of Planck time from stellar interferometry*, astro-ph/0211402.
- [47] Giovanni Amelino-Camelia, *An interferometric gravitational wave detector as a quantum-gravity apparatus*, gr-qc/9808029, Nature 398 (1999) 216-218.

- [48] Robert H. Brandenberger and Jerome Martin *On Signatures of Short Distance Physics in the Cosmic Microwave Background*, hep-th/0202142, Int.J.Mod.Phys. A17 (2002) 3663-3680.
- [49] J. K. Webb, M. T. Murphy, V. V. Flambaum, S. J. Curran, *Does the fine structure constant vary? A third quasar absorption sample consistent with varying alpha*, astro-ph/0210531, ApSS, 283, 565, 200; M. T. Murphy, J. K. Webb, V. V. Flambaum, S. J. Curran, *Does the fine structure constant vary? A detailed investigation into systematic effects*, astro-ph/0210532, ApSS, 283, 577, 2003; John N. Bahcall, Charles L. Steinhardt, David Schlegel, *Does the fine-structure constant vary with cosmological epoch?*, astro-ph/0301507.
- [50] Roger Penrose, *The emperor's new mind : concerning computers, minds, and the laws of physics* Oxford Univ. Press, 1989.
- [51] Gerard 't Hooft, *Quantum Mechanics and Determinism*, hep-th/0105105; *How Does God Play Dice? (Pre-)Determinism at the Planck Scale*, hep-th/0104219.
- [52] L. Bombelli, J. H. Lee, D. Meyer and R. Sorkin, "Space-Time As A Causal Set," Phys. Rev. Lett. **59**, 521 (1987); X. Martin, D. O'Connor, D. P. Rideout and R. D. Sorkin, "On the 'renormalization' transformations induced by cycles of expansion and contraction in causal set cosmology," Phys. Rev. D **63**, 084026 (2001) [arXiv:gr-qc/0009063]; D. P. Rideout and R. D. Sorkin, "Evidence for a continuum limit in causal set dynamics," Phys. Rev. D **63**, 104011 (2001) [arXiv:gr-qc/0003117].
- [53] J. Ambjorn, Z. Burda, J. Jurkiewicz and C. F. Kristjansen, "Quantum gravity represented as dynamical triangulations," Acta Phys. Polon. B **23**, 991 (1992); J. Ambjorn, "Quantum Gravity Represented As Dynamical Triangulations," Class. Quant. Grav. **12**, 2079 (1995); M. E. Agishtein and A. A. Migdal, "Simulations of four-dimensional simplicial quantum gravity," Mod. Phys. Lett. A **7**, 1039 (1992).
- [54] J. Ambjorn, A. Dasgupta, J. Jurkiewicz and R. Loll, "A Lorentzian cure for Euclidean troubles", hep-th/0201104; J. Ambjorn and R. Loll, Nucl. Phys. B536 (1998) 407 [hep-th/9805108]; J. Ambjorn, J. Jurkiewicz and R. Loll, Phys. Rev. Lett. 85 (2000) 924 [hep-th/0002050]; Nucl. Phys. B610 (2001) 347 [hep-th/0105267]; R. Loll, Nucl. Phys. B (Proc. Suppl.) 94 (2001) 96 [hep-th/0011194]; J. Ambjorn, J. Jurkiewicz and R. Loll, Phys. Rev. D64 (2001) 044011 [hep-th/0011276]; JHEP 09 (2001) 022 [hep-th/0106082]; B. Dittrich (AEI, Golm), R. Loll, *A Hexagon Model for 3D Lorentzian Quantum Cosmology*, hep-th/0204210.
- [55] A. Dasgupta and R. Loll, Nucl. Phys. B606 (2001) 357 [hep-th/0103186].
- [56] J. Ambjorn (Niels Bohr Institute), K.N. Anagnostopoulos (Univ. of Crete), R. Loll, *Crossing the $c=1$ barrier in 2d Lorentzian quantum gravity*, hep-lat/9909129, Phys.Rev. D61 (2000) 044010.

- [57] R. Penrose and M. A. MacCallum, “Twistor Theory: An Approach To The Quantization Of Fields And Space-Time,” *Phys. Rept.* **6**, 241 (1972); R. Penrose, “Twistor Theory, Its Aims And Achievements,” *In *Chilton 1974, Proceedings, Oxford Symposium On Quantum Gravity**, Oxford 1975, 268-407.
- [58] Alain Connes, *Noncommutative geometry*. Academic Press, Inc., San Diego, CA, 1994. xiv+661 pp.
- [59] See, for example, Earman, J. 1989. *World Enough and Spacetime: Absolute vs. Relational Theories of Space and Time*. Cambridge, MA: MIT Press; Norton, J. D. 1987. Einstein, the Hole Argument and the Reality of Space. In J. Forge (ed.), *Measurement, Realism and Objectivity*, 153-188. Boston: D. Reidel; L. Smolin, “The present moment in quantum cosmology: Challenges to the arguments for the elimination of time,” arXiv:gr-qc/0104097.
- [60] P. A. M. Dirac, *Lectures on Quantum Mechanics* Belfer Graduate School of Science Monographs, no. 2 (Yeshiva University Press, New York, 1964).
- [61] J. Stachel, “Einstein’s search for general covariance, 1912-15” in *Einstein and the History of General Relativity* vol 1 of *Einstein Studies* eds. D. Howard and J. Stachel. (Birkhauser, Boston, 1989).
- [62] Julian Barbour, *The End of Time*, Oxford University Press, 2000; “Leibnizian Time, Machian Dynamics, And Quantum Gravity,” *In *Oxford 1984, Proceedings, Quantum Concepts In Space and Time**, 236-246.
- [63] C. Rovelli, “What Is Observable In Classical And Quantum Gravity?,” *Class. Quant. Grav.* **8**, 297 (1991).
- [64] L. Smolin *Life of the Cosmos*, (Oxford University Press, New York and Wiedenfeld and Nicolson, London, 1997).
- [65] R. De Pietri, L. Freidel, K. Krasnov, C. Rovelli, Barrett-Crane model from a Boulatov-Ooguri theory over a homogeneous space, *Nucl.Phys.* B574 (2000) 785-806, hep-th/9907154.
- [66] T. Banks, W. Fischler, S. H. Shenker, and L. Susskind, M theory as a matrix model: A conjecture,” *Phys. Rev.* D55 (1997) 51125128, hep-th/9610043; M. Claudson and M. Halpern, *Nucl. Phys.* B250 (1985) 689.
- [67] N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, A large-n reduced model as superstring,” *Nucl. Phys.* B498 (1997) 467491, hep-th/9612115.
- [68] L. Crane in, *Categorical Physics*, hep-th/9301061; Topological Field theory as the key to quantum gravity, hep-th/9308126, in *Knot theory and quantum gravity* ed. J. Baez,

- (Oxford University Press); Clocks and Categories, is quantum gravity algebraic? *J. Math. Phys.* 36 (1995) 6180-6193, gr-qc/9504038.
- [69] G. 'tHooft, Dimensional reduction in quantum gravity gr-qc/9310006.in Salanfestschrift ed. A. Alo, J. Ellis, S. Randjbar-Daemi (World Scientific,1993).
- [70] L. Susskind, The world as a hologram, hep-th/9409089, *J. Math. Phys.* 36 (1995) 6377 ; Strings, black holes and Lorentz contractions hep-th/ 9308139; *Phys. Rev. Lett.* 71 (1993) 2367; *Phys. Rev. D* 49 (1994) 6606; *D* 50 (1994) 2700; L. Susskind and P. Gri n, Partons and black holes hep-th/9410306.
- [71] F. Markopoulou and L. Smolin, Holography in a quantum spacetime, hep-th/9910146.
- [72] L. Smolin, The strong and the weak holographic principles, hep-th/0003056.
- [73] R. Loll, "Spectrum of the Volume Operator in Quantum Gravity," *Nucl. Phys. B* **460**, 143 (1996) [arXiv:gr-qc/9511030]; "The Volume operator in discretized quantum gravity," *Phys. Rev. Lett.* **75**, 3048 (1995) [arXiv:gr-qc/9506014].
- [74] Roberto De Pietri and Carlo Rovelli, Geometry eigenvalues and the scalar product from recoupling theory in loop quantum gravity," *Phys. Rev. D* 54(4), 2664-2690 (1996); S. Frittelli, L. Lehner, C. Rovelli, The complete spectrum of the area from recoupling theory in loop quantum gravity," *Class. Quant. Grav.* 13, 2921-2932 (1996); Abhay Ashtekar and Jerzy Lewandowski, Quantum Theory of Geometry I: Area operators," *Class. Quant. Grav.* 14, A55-A81 (1997).
- [75] S. A. Major, "Operators for quantized directions," *Class. Quant. Grav.* **16**, 3859 (1999) [arXiv:gr-qc/9905019]; "New operators for spin net gravity: Definitions and consequences," arXiv:gr-qc/0101032.
- [76] J.F. Plebanski. On the separation of einsteinian substructures. *J. Math. Phys.*, 18:2511, 1977.
- [77] T. Jacobson and L. Smolin, *Phys. Lett. B* 196 (1987) 39; *Class. and Quant. Grav.* 5 (1988) 583; J. Samuel, *Pramana-J Phys.* 28 (1987) L429.
- [78] R. Capovilla, J. Dell and T. Jacobson, *Phys. Rev. Lett.* 21, 2325(1989); *Class. Quant. Grav.* 8, 59(1991); R. Capovilla, J. Dell, T. Jacobson and L. Mason, *Class. and Quant. Grav.* 8, 41(1991).
- [79] R. Penrose, Theory of quantized directions unpublished manuscript; in Quantum theory and beyond ed T Bastin, Cambridge U Press 1971, in *Advances in Twistor Theory*, ed. L. P. Hughston and R. S. Ward, (Pitman,1979) p. 301in *Combinatorial Mathematics and its Application* (ed. D. J. A. Welsh) (Academic Press,1971).

- [80] C. Rovelli and L. Smolin, The physical hamiltonian in nonperturbative quantum gravity, Phys. Rev. Lett.72(1994)446; Spin Networks and Quantum Gravity, Phys. Rev. D52(1995)5743-5759.
- [81] R. Borissov, Graphical Evolution of Spin Network States, Phys.Rev. D55 (1997) 6099-6111, gr-qc/9606013.
- [82] T. Thiemann, Quantum Spin Dynamics (QSD) I & II, Class.Quant.Grav. 15 (1998) 839-905, gr-qc/9606089, gr-qc/9606090.
- [83] L. Smolin, *Time, measurement and information loss in quantum cosmology*, gr-qc/9301016.
- [84] L. Smolin, *Finite, diffeomorphism invariant observables in quantum gravity*, Phys. Rev. D 49 (1994) 4028, gr-qc/9302011.
- [85] M. P. Reisenberger. “Worldsheet formulations of gauge theories and gravity,” in Proceedings of the 7th Marcel Grossman Meeting, ed. by R. Jantzen and G. MacKeiser, World Scientific, 1996; gr-qc/9412035; A lattice worldsheet sum for 4-d Euclidean general relativity. gr-qc/9711052.
- [86] 8] M. P. Reisenberger and C. Rovelli. Sum-over-surface form of loop quantum gravity,” gr-qc/9612035, Phys. Rev. D 56 (1997) 3490; Spacetime as a Feynman diagram: the connection formulation. Class.Quant.Grav., 18:121140, 2001; Spin foams as Feynman diagrams, gr-qc/0002083.
- [87] J. Barrett and L. Crane, “Relativistic spin networks and quantum gravity”, J. Math. Phys. 39 (1998) 3296-3302, gr-qc/9709028.
- [88] J. Baez, Spin foam models, Class. Quant. Grav. 15 (1998) 1827-1858, gr-qc/9709052; An introduction to spin foam models of quantum gravity and BF theory. Lect.Notes Phys., 543:25 94, 2000.
- [89] Fotini Markopoulou, “Dual formulation of spin network evolution”, gr-qc/9704013.
- [90] F. Markopoulou, *Planck-scale models of the Universe*, gr-qc/0210086.
- [91] E. Hawkins, F. Markopoulou, H. Sahlmann, *Evolution in Quantum Causal Histories*, hep-th/0302111.
- [92] Fotini Markopoulou, Lee Smolin, “Quantum geometry with intrinsic local causality”, Phys.Rev. D58 (1998) 084032, gr-qc/9712067.
- [93] Fotini Markopoulou, Lee Smolin, *Causal evolution of spin networks*, gr-qc/9702025, Nucl.Phys. B508 (1997) 409-430.

- [94] Stuart Kauffman, Lee Smolin, “Combinatorial dynamics in quantum gravity”, hep-th/9809161, Lect.Notes Phys. 541 (2000) 101-129.
- [95] David Rideout, *Dynamics of Causal Sets*, gr-qc/0212064; D. P. Rideout, R. D. Sorkin, *A Classical Sequential Growth Dynamics for Causal Sets*, gr-qc/9904062, Phys.Rev. D61 (2000) 024002; *Evidence for a continuum limit in causal set dynamics*, gr-qc/0003117, Phys.Rev. D63 (2001) 104011.
- [96] J. Iwasaki, A reformulation of the Ponzano-Regge quantum gravity model in terms of surfaces,” gr-qc/9410010; A definition of the Ponzano-Regge quantum gravity model in terms of surfaces,” gr-qc/9505043, J. Math. Phys. 36 (1995) 6288; L. Freidel and K. Krasnov. Spin foam models and the classical action principle. Adv.Theor.Math.Phys., 2:11831247, 1999; R. De Pietri, L. Freidel, K. Krasnov, and C. Rovelli. Barrett-Crane model from a boulatov-ooguri field theory over a homogeneous space. Nucl.Phys. B, 574:785806, 2000.
- [97] T. Thiemann, “Quantum spin dynamics,” gr-qc/9606089, gr-qc/9606090; “Anomaly free formulation of non-perturbative 4-dimensional Lorentzian quantum gravity,” Phys. Lett. B380 (1996) 257.
- [98] L. Smolin, Linking topological quantum field theory and nonperturbative quantum gravity, J. Math. Phys. 36(1995)6417, gr-qc/9505028.
- [99] K. Krasnov, On Quantum Statistical Mechanics of a Schwarzschild Black Hole , grqc/9605047, Gen. Rel. Grav. 30 (1998) 53-68; C. Rovelli, “Black hole entropy from loop quantum gravity,” grqc/9603063.
- [100] A. Ashtekar, J. Baez, K. Krasnov, Quantum Geometry of Isolated Horizons and Black Hole Entropy gr-qc/0005126; A. Ashtekar, J. Baez, A. Corichi, K. Krasnov, “Quantum geometry and black hole entropy,” gr-qc/9710007, Phys.Rev.Lett. 80 (1998) 904-907.
- [101] L. Smolin, A holographic formulation of quantum general relativity, Phys. Rev. D61 (2000) 084007, hep-th/9808191.
- [102] J. D. Bekenstein, Lett. Nuovo. Cim 11(1974).
- [103] S. Das, Parthasarathi Majumdar, Rajat K. Bhaduri, “ General Logarithmic Corrections to Black Hole Entropy”, Class.Quant.Grav. 19 (2002) 2355-2368; Saurya Das, hep-th/0207072, “Leading Log Corrections to Bekenstein-Hawking Entropy”
- [104] Kirill Krasnov, “Quantum Geometry and Thermal Radiation from Black Holes”, gr-qc/9710006, Class.Quant.Grav. 16 (1999) 563-578.
- [105] M. Barreira, M. Carfora, C. Rovelli, “Physics with nonperturbative quantum gravity: radiation from a quantum black hole” gr-qc/9603064, Gen.Rel.Grav. 28 (1996) 1293-1299.

- [106] M. Atiyah, Topological quantum field theory Publ. Math. IHES 68 (1989) 175; The Geometry and Physics of Knots, Lezion Lincee (Cambridge University Press, Cambridge,1990); G. Segal, Conformal field theory oxford preprint (1988).
- [107] G. Horowitz, “Exactly Soluble Diffeomorphism Invariant Theories”, Commun.Math.Phys. 125 (1989) 417; V. Husain, “Topological Quantum Mechanics”, Phys.Rev. D43 (1991) 1803.
- [108] H. Kodama, Prog. Theor. Phys. 80, 1024(1988); Phys. Rev. D42(1990)2548.
- [109] L. Smolin and C. Soo, The Chern-Simons Invariant as the Natural Time Variable for Classical and Quantum Cosmology, Nucl. Phys. B449 (1995) 289, gr-qc/9405015.
- [110] L. N. Chang and C. Soo, Ashtekar’s variables and the topological phase of quantum gravity, Proceedings of the XXth. Conference on Differential Geometric Methods in Physics, (Baruch College, New York, 1991), edited by S. Catto and A. Rocha (World Scientific, 1992); Phys. Rev. D46 (1992) 4257; C. Soo and L. N. Chang, Int. J. Mod. Phys. D3 (1994) 529.
- [111] C. Soo, Wave function of the Universe and Chern-Simons Perturbation Theory, gr-qc/0109046.
- [112] L. Smolin, The classical limit and the form of the hamiltonian constraint in nonperturbative quantum gravity, gr-qc/9607034.
- [113] J. Lewandowski, D. Marolf, Loop constraints: a habitat and their algebra,” gr-qc/9710016; R. Gambini, J. Lewandowski, D. Marolf, J. Pullin, “On the consistency of the constraint algebra in spin-network quantum gravity,” gr-qc/9710018.
- [114] A. Ashtekar, C. Rovelli, L. Smolin, Weaving a classical metric with quantum threads,” Phys. Rev. Lett. 69 (1992) 237.
- [115] Luca Bombelli, Statistical geometry of random weave states, gr-qc/0101080
- [116] J. Iwasaki, C. Rovelli, Gravitons as embroidery on the weave,” Int. J. Mod. Phys. D 1 (1993) 533; Gravitons from loops: non-perturbative loop-space quantum gravity contains the graviton-physics approximation,” Class. Quantum Grav. 11 (1994) 1653.
- [117] Martin Bojowald,”Isotropic Loop Quantum Cosmology”, Class.Quant.Grav. 19 (2002) 2717-2742, gr-qc/0202077; “Inflation from Quantum Geometry”, gr-qc/0206054; “The Semiclassical Limit of Loop Quantum Cosmology”, gr-qc/0105113, Class.Quant.Grav. 18 (2001) L109-L116; “ Dynamical Initial Conditions in Quantum Cosmology”, gr-qc/0104072, Phys.Rev.Lett. 87 (2001) 121301.

- [118] A. Ashtekar, V. Husain, C. Rovelli, J. Samuel and L. Smolin 2+1 quantum gravity as a toy model for the 3+1 theory *Class. and Quantum Grav.* L185- L193 (1989); L. Smolin, Loop representation for quantum gravity in 2+1 dimensions, in the proceedings of the John's Hopkins Conference on Knots, Topology and Quantum Field Theory ed. L. Lusanna (World Scientific, Singapore, 1989) .
- [119] Vigar Husain, "Observables for spacetimes with two Killing field symmetries", gr-qc/9402019, *Phys. Rev. D* 50 (1994) 6207-6216; V. Husain and L. Smolin, "Exactly Solvable Quantum Cosmologies From Two Killing Field spacetimes", *Nucl. Phys. B* 327 (1989) 205.
- [120] R. Gianvittorio, R. Gambini and A. Trias, *Phys. Rev. D* 38 (1988) 702; C. Rovelli and L. Smolin. Loop representation for lattice gauge theory, 1990 Pittsburgh and Syracuse preprint; B. Bruegmann, *Physical Review D* 43 (1991) 566; J.M.A. Farrerons, Loop calculus for SU(3) on the lattice Phd. thesis, Universitat Autònoma de Barcelona (1990); R. Loll A new quantum representation for canonical gravity and SU(2) Yang-Mills theory, University of Bonn preprint, BONN-HE-90-02 (1990)
- [121] A Ashtekar J Lewandowski D Marlof J Mourau T Thiemann: "Quantization of diffeomorphism invariant theories of connections with local degrees of freedom", gr-qc/9504018, *JMP* 36 (1995) 519.
- [122] Thomas Thiemann, "Introduction to Modern Canonical Quantum General Relativity" gr-qc/0110034.
- [123] J. Barbero, Real Ashtekar variables for Lorentzian signature spacetime," *Phys. Rev. D* 51 (1995) 5507.
- [124] Laurent Freidel, Kirill Krasnov, Discrete Space-Time Volume for 3-Dimensional BF Theory and Quantum Gravity, *Class.Quant.Grav.* 16 (1999) 351-362.
- [125] Fotini Markopoulou, "Coarse graining in spin foam models", gr-qc/0203036;
- [126] Fotini Markopoulou, "An algebraic approach to coarse graining", hep-th/0006199.
- [127] Alejandro Perez, Finiteness of a spinfoam model for euclidean quantum general relativity, *Nucl. Phys. B* 599 (2001) 427-434, gr-qc/0011058 ; Louis Crane, Alejandro Perez, Carlo Rovelli, A finiteness proof for the Lorentzian state sum spinfoam model for quantum general relativity, gr-qc/0104057.
- [128] John C. Baez, J. Daniel Christensen, Positivity of Spin Foam Amplitudes , *Class.Quant.Grav.* 19 (2002) 2291-2306, gr-qc/0110044; John C. Baez, J. Daniel Christensen, Greg Egan, Asymptotics of 10j symbols , gr-qc/0208010.

- [129] H. Sahlmann, T. Thiemann, O. Winkler, Coherent States for Canonical Quantum General Relativity and the Infinite Tensor Product Extension, Nucl.Phys. B606 (2001) 401-440, gr-qc/0102038.
- [130] Eric D'Hoker and D.H. Phong, *TWO-LOOP SUPERSTRINGS I* Phys.Lett. B529 (2002) 241-255, hep-th/0110247; *Lectures on Two-Loop Superstrings*, hep-th/0211111.
- [131] K. Aoki, E. D'Hoker and D. H. Phong, "Unitarity Of Closed Superstring Perturbation Theory," Nucl. Phys. B **342**, 149 (1990).
- [132] Howe and Stelle, Int.J.Mod.Phys.A4:1871,1989; see also Zvi Ber, Perturbative Quantum Gravity and its Relation to Gauge Theory, gr-qc/0206071
- [133] D. J. Gross and V. Periwal, "String Perturbation Theory Diverges," Phys. Rev. Lett. **60**, 2105 (1988)
- [134] S. Kachru, J. Kumar, and E. Silverstein, Vacuum energy cancellation in a non-supersymmetric string", Phys. Rev D59 (1999) 106004, hep-th/9807076; S. Kachru and E. Silverstein, "Self-dual nonsupersymmetric Type II string compactifications", JHEP 9811:001 (1998) hep-th/9808056; S. Kachru and E. Silverstein, "On vanishing two loop cosmological constants in nonsupersymmetric strings", JHEP 9901:004 (1999) hep-th/9810129; R. Iengo and C.J. Zhu, "Evidence for Non-Vanishing Cosmological Constant in Non- SUSY Superstring Models", JHEP 0004:028 (2000), hep-th/9912074.
- [135] D. Friedan, "On Two-Dimensional Conformal Invariance And The Field Theory Of String," Phys. Lett. B **162**, 102 (1985); C. G. Callan, E. J. Martinec, M. J. Perry and D. Friedan, "Strings In Background Fields," Nucl. Phys. B **262**, 593 (1985).
- [136] J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, astro-ph/0108295, Phys.Rev. D65 (2002) 064007.
- [137] Brian R. Greene, David R. Morrison, Andrew Strominger, Black Hole Condensation and the Unification of String Vacua, Nucl.Phys. B451 (1995) 109-120, hep-th/9504145.
- [138] A. Klemm and R. Schimmrigk, Landau- Ginzburg string vacua," Nucl. Phys. B411 (1994) 559; Christof Schmidhuber, *Old Puzzles*, hep-th/0207203
- [139] E. Witten, *Quantum gravity in de Sitter space*, hep-th/0106109; Quantum gravity in de Sitter space," Strings 2001 online proceedings <http://theory.tifr.res.in/strings/Proceedings>; One aspect of the problem is that there are no unitary representations of the supersymmetric extension of the deSitter group, see K. Pilch, P. van Nieuwenhuizen and M. F. Sohnius, De Sitter Superalgebras And Supergravity," Commun. Math. Phys. 98, 105 (1985).
- [140] M. Spradlin, A. Strominger and A. Volovich1y Les Houches Lectures on de Sitter Space, hep-th/0110007.

- [141] Joseph Polchinski, Dirichlet-Branes and Ramond-Ramond Charges, Phys.Rev.Lett. 75 (1995) 4724-4727, hep-th/9510017.
- [142] Joseph Polchinski, Shyamoli Chaudhuri, Clifford V. Johnson, Notes on D-Branes, hep-th/9602052; Clifford Johnson, *D Branes* Cambridge University press, 2002, in Press.
- [143] Mirjam Cvetič, Paul Langacker, Gary Shiu, Phenomenology of A Three-Family Standard-like String Model hep-ph/0205252.
- [144] Amanda W. Peet, TASI lectures on black holes in string theory, hep-th/0008241
- [145] A. Strominger and C. Vafa, Microscopic Origin of the Bekenstein-Hawking Entropy,” Phys. Lett. B 379, 99 (1996) [hep-th/9601029].
- [146] For a recent review, see Sumit R. Das and Samir D. Mathur, *THE QUANTUM PHYSICS OF BLACK HOLES: Results from String Theory*, hep-th/0105063.
- [147] J.C. Breckenridge, D.A. Lowe, R.C. Myers, A.W. Peet, A. Strominger, C. Vafa, Macroscopic and Microscopic Entropy of Near-Extremal Spinning Black Holes, hep-th/9603078, Phys.Lett. B381 (1996) 423-426.
- [148] For a review, see, Jerome P. Gauntlett Duality and Supersymmetric Monopoles, hep-th/9705025
- [149] A. Sen, Phys. Lett. B329 (1994) 217; C. Vafa and E. Witten, *A Strong Coupling Test of S-Duality*, Nucl.Phys. B431 (1994) 3-77, hep-th/9408074.
- [150] Ashoke Sen, Non-BPS States and Branes in String Theory, hep-th/9904207.
- [151] A. Polyakov, The wall of the cave, Int.J.Mod.Phys. A14 (1999) 645-658, hep-th/9809057.
- [152] J. Maldacena, The large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2 (1998) 231 [hep-th/9711200].
- [153] E.Witten, Anti-de Sitter space and holography,” Adv. Theor.Math. Phys. 2 (1998) 253 [hep-th/9802150].
- [154] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Gauge theory correlators from non-critical string theory,” Phys. Lett. B 428 (1998) 105 [hep-th/9802109]
- [155] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, Large N eld theories, string theory and gravity,” Phys. Rept. 323 (2000) 183 [hep-th/9905111].
- [156] M. Arnsdorf and L. Smolin, hep-th/0106073, *The Maldacena conjecture and Rehren duality*

- [157] Edward Witten, Anti-de Sitter Space, Thermal Phase Transition, And Confinement In Gauge Theories, hep-th/9803131, Adv.Theor.Math.Phys. 2 (1998) 505-532.
- [158] A. A. Tseytlin, Superstrings in AdS in light cone gauge,” hep-th/0009226.
- [159] R.R. Metsaev and A.A. Tseytlin, Type IIB superstring action in $AdS^5 \times S^5$ background,” Nucl. Phys. B533, 109 (1998), hep-th/9805028.
- [160] N. Drukker, D. J. Gross and A. A. Tseytlin, *Green-Schwarz String in $AdS^5 \times S^5$: Semiclassical Partition Function*, hep-th/0001204
- [161] Renata Kallosh and Arvind Rajaraman, VACUA OF M-THEORY AND STRING THEORY, hep-th/9805041.
- [162] D. Berenstein, J. M. Maldacena, and H. Nastase, Strings in at space and pp waves from $n = 4$ super yang mills,” JHEP 04 (2002) 013, hep-th/0202021.
- [163] K. H. Rehren, Algebraic holography,” Annales Henri Poincare 1 (2000) 607 [hep-th/9905179]. [6] K.-H. Rehren, Local quantum observables in the anti-deSitter - conformal QFT correspondence,” Phys. Lett. B493 (2000) 383 [hep-th/0003120]. [7] K. H. Rehren, A proof of the AdS/CFT correspondence,” hep-th/9910074.
- [164] M. Henningson and K. Skenderis, The holographic Weyl anomaly,” JHEP 9807, 023 (1998) [hep-th/9806087].
- [165] L. Smolin, ‘Did The Universe Evolve?,” Class. Quant. Grav. **9**, 173 (1992); “The Fate of black hole singularities and the parameters of the standard models of particle physics and cosmology,” arXiv:gr-qc/9404011; “Cosmology as a problem in critical phenomena,” arXiv:gr-qc/9505022.
- [166] D.H. Wolpert and W.G. Macready, *No Free Lunch Theorems for Optimization*, IEEE Transactions on Evolutionary Computation 1, vol. 1, 1997, pp. 67-82.
- [167] M. Li,
E. Martinec, *Black Holes in Matrix Theory*, hep-th/9709114, Nucl.Phys.Proc.Suppl. 68 (1998) 329-335; Igor R. Klebanov, Leonard Susskind, *Schwarzschild Black Holes in Various Dimensions from Matrix Theory* 20. hep-th/9709108, Phys.Lett. B416 (1998) 62-66; T.Banks, W.Fischler, I.R.Klebanov, L.Susskind, *Schwarzschild Black Holes from Matrix Theory*, hep-th/9709091, Phys.Rev.Lett. 80 (1998) 226-229.
- [168] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 429, 263 (1998) [hep-ph/9803315]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 436, 257 (1998) [hep-ph/9804398]; L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 4690 (1999) [hep-th/9906064]; Phys. Rev. Lett. 83, 3370 (1999) [hep-ph/9905221].

- [169] C. M. Hull and P. K. Townsend, Unity of superstring dualities, Nucl. Phys. B348 (1995) 109; P. Townsend, (M)embrane theory on T 9, Nucl. Phys. (Proc. Suppl) 68 (1998) 11-16; hep-th/9507048, in *Particles, Strings and Cosmology*, ed. J. Bagger et al (World Scientific, 1996); hep-th/9612121; I. Bars, hep-th/9608061; hep-th/9607122; Petr Horava, MTheory as a Holographic Field Theory, hep-th/9712130, Phys.Rev. D59 (1999) 046004.
- [170] E. Witten, String theory in various dimensions, Nucl. Phys. B443 (1995) 85.
- [171] E. Cremmer, B. Julia and J. Scherk, Phys. Lett. B76, 409 (1978).
- [172] B. DeWitt, J. Hoppe, H. Nicolai, Nuclear Physics B305 (1988) 545.
- [173] Lee Smolin, *Strings as perturbations of evolving spin-networks*, Nucl.Phys.Proc.Suppl. 88 (2000) 103-113, hep-th/9801022.
- [174] L. Smolin, “M theory as a matrix extension of Chern-Simons theory”, hep-th/0002009, Nucl.Phys. B591 (2000) 227-242; “The cubic matrix model and a duality between strings and loops”, hep-th/0006137; “The exceptional Jordan algebra and the matrix string”, hep-th/0104050.
- [175] Takehiro Azuma, Maxime Bagnoud, Curved-space classical solutions of a massive supermatrix model, hep-th/0209057; Maxime Bagnoud, Luca Carlevaro, Adel Bilal, Supermatrix models for M-theory based on $osp(1-32, R)$, Nucl.Phys.B641:61-92,2002, hep-th/0201183; Takehiro Azuma, Satoshi Iso, Hikaru Kawai, Yuhi Ohwashi, Supermatrix Models, Nucl.Phys. B610 (2001) 251-279, hep-th/0102168; Takehiro Azuma, Investigation of Matrix Theory via Super Lie Algebra, hep-th/0103003.
- [176] L. Susskind, L. Thorlacius and J. Uglum, “The Stretched horizon and black hole complementarity,” Phys. Rev. D **48**, 3743 (1993) [arXiv:hep-th/9306069].
- [177] Carlo Rovelli, “Relational Quantum Mechanics” quant-ph/9609002, Int. J. of Theor. Phys. 35 (1996) 1637.
- [178] L. Smolin, The Bekenstein Bound, Topological Quantum Field Theory and Pluralistic Quantum Field Theory, gr-qc/9508064.
- [179] Fotini Markopoulou, “An insider’s guide to quantum causal histories”, hep-th/9912137, Nucl.Phys.Proc.Suppl. 88 (2000) 308-313; “Quantum causal histories”, hep-th/9904009, Class.Quant.Grav. 17 (2000) 2059-2072; “The internal description of a causal set: What the universe looks like from the inside”, Commun.Math.Phys. 211 (2000) 559-583, gr-qc/9811053; E. Hawkins, F. Markopoulou, H. Sahlmann, *Evolution in quantum causal histories*, hep-th/0302111.
- [180] C.J. Isham, J. Butterfield, “Some Possible Roles for Topos Theory in Quantum Theory and Quantum Gravity”, gr-qc/9910005, Found.Phys. 30 (2000) 1707-1735; “A topos

- perspective on the Kochen-Specker theorem: II. Conceptual Aspects, and Classical Analogues” quant-ph/9808067, Int.J.Theor.Phys. 38 (1999) 827-859; “A topos perspective on the Kochen-Specker theorem: I. Quantum States as Generalized Valuations”, quant-ph/9803055; C.J. Isham, “Topos Theory and Consistent Histories: The Internal Logic of the Set of all Consistent Sets”, gr-qc/9607069, Int.J.Theor.Phys. 36 (1997) 785-814.
- [181] P. Horava, M-Theory as a Holographic Field Theory, Phys.Rev. D59 (1999) 046004, hep-th/9712130.
- [182] M. Banados, M. Henneaux, C. Iannuzzo and C. M. Viallet, A note on the gauge symmetries of pure Chern-Simons theory with p-form gauge elds gr-qc/9703061; Max Banados, Luis J. Garay and Marc Henneaux, Nucl.Phys.B476:611-635,1996, hep-th/9605159; Phys.Rev.D53:593-596,1996, hep-th/9506187.
- [183] R. Floreanini and R. Percacci, Phys. Lett. B224 (1989) 291-294; B231:119-124,1989. [35] V.V. Fock, N.A. Nekrasov, A.A. Rosly, K.G. Selivanov What we think about the higher dimensional Chern-Simons theories (Moscow, ITEP). ITEP-91-70, July 1991. 7pp. in Sakharov Conf.1991:465-472 (QC20:I475:1991)
- [184] T.Banks, “Cosmological Breaking of Supersymmetry?”, hep-th/0007146.
- [185] T. Banks and W. Fischler, *M-theory observables for cosmological space-times* hep-th/0102077.
- [186] L. Freidel and David Louapre, *Non-perturbative summation over 3D discrete topologies*, hep-th/0211026.
- [187] R. Dijkgraaf, C. Vafa, *A Perturbative Window into Non-Perturbative Physics*, hep-th/0208048.
- [188] A Marshakov hep-th/0212114
- [189] Stefano Bellucci, Corneliu Sochichiu, *Fermion Doubling and Berenstein–Maldacena–Nastase Correspondence*, hep-th/0302104
- [190] O. Dreyer, *Quasinormal Modes, the Area Spectrum, and Black Hole Entropy*, gr-qc/0211076
- [191] L. Motl, *An analytical computation of asymptotic Schwarzschild quasinormal frequencies*, gr-qc/0212096; L. Motl, A. Neitzke, *Asymptotic black hole quasinormal frequencies*, hep-th/0301173.
- [192] S. Kachru, R. Kallosh, A. Linde, S. P. Trivedi, *de Sitter Vacua in String Theory* hep-th/0301240.
- [193] L. Susskind, *The Anthropic Landscape of String Theory*, hep-th/0302219.

- [194] H.S.Egawa, S.Horata, T.Yukawa, *Clear Evidence of a Continuum Theory of 4D Euclidean Simplicial Quantum Gravity, Further Evidence of a Smooth Phase in 4D Simplicial Quantum Gravity*, Nucl.Phys.Proc.Suppl. 73 (1999) 795-797, hep-lat/0110042.
- [195] G Ponzano, T Regge, *Semiclassical limits of Racah Coefficients in Spectroscopy and Group theoretical methods in Physics*. Ed F Bloch, North Holland, Amsterdam 1968
- [196] W. Beirl, E. Gerstenmayer, H. Markum, *Influence of the Measure on Simplicial Quantum Gravity in Four Dimensions*, hep-lat/9204010, Phys.Rev.Lett. 69 (1992) 713-716; Elmar Bittner, Wolfhard Janke, Harald Markum, *On the Continuum Limit of the Discrete Regge Model in 4d*, hep-lat/0209028; W. Beirl, A. Hauke, P. Homolka, B. Krishnan, H. Markum and J. Riedler, Nucl. Phys. B (Proc. Suppl.) 47 (1996) 625; J. Riedler, W. Beirl, E. Bittner, A. Hauke, P. Homolka and H. Markum, Class. Quant. Grav. 16 (1999) 1163.
- [197] A. Connes, *Noncommutative Geometry*, Academic Press, 1994.
- [198] E. Hawkins, *Noncommutative Rigidity*, math.QA/0211203.
- [199] Maqbool Ahmed, Scott Dodelson, Patrick B. Greene, Rafael Sorkin, *Everpresent Lambda*, astro-ph/0209274.
- [200] R. Penrose, *Twistor Theory: An Approach To The Quantization Of Fields And Space-Time*, Phys.Rept. 6 (1972) 241
- [201] P. Claus, M. Gunaydin, R. Kallosh, J. Rahmfeld, Y. Zunger, *Supertwistors as Quarks of $SU(2,2-4)$* , hep-th/9905112, JHEP 9905 (1999) 019.
- [202] G.E. Volovik, *Superfluid analogies of cosmological phenomena*, gr-qc/0005091.
- [203] R. B. Laughlin, *Emergent Relativity*, gr-qc/0302028.
- [204] Shou-Cheng Zhang, *To see a world in a grain of sand*, hep-th/0210162; Shou-Cheng Zhang, Jiangping Hu, *A Four Dimensional Generalization of the Quantum Hall Effect*, cond-mat/0110572, Science 294 (2001) 823.
- [205] K. Agashe, A. Delgado and R. Sundrum, *Grand unification in RS1*, hep-ph/0212028
- [206] R. Dermisek, A. Mafi and S. Raby, *SUSY GUTS under siege: proton decay*, hep-ph/0007213
- [207] L. Crane, *An Octonionic Geometric (Balanced) state Sum Model*, gr-qc/9806060.
- [208] L. Crane, *A new approach to the geometrization of matter*, gr-qc/0110060.